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RESEARCH ARTICLE

Penumbral connections in comparative constructions

Heather Burnett*

University of Montreal, Quebec, Canada

This paper gives a novel analysis of the logical structure underlying three classes of vague adjectival predicates (relative adjectives, i.e., *tall*; total adjectives, i.e., *straight*; and partial adjectives, i.e., *wet*) and the realisation of this structure in arguments formed with comparative constructions (i.e., *John is taller than Mary*). I analyse three classes of valid arguments that can be formed with different types of gradable predicates in comparative constructions: *scalarity* arguments (i.e., *Mary is taller than John and John is tall* ∴ *Mary is tall*), *maximality* arguments (i.e., *Stick A is straighter than Stick B* ∴ *Stick B is not straight*), and *evaluativity* arguments (i.e., *This towel is wetter than that towel* ∴ *this towel is wet*) I show how a previously proposed multi-valued logical system (called Delineation TCS; see Burnett, 2012b, 2013) based on the trivalent framework of Cobreros, Égré, Ripley and van Rooij (2012) can be used to model the reasoning patterns described in the paper. In this system, we derive the scalarity, maximality and evaluativity properties of different classes of adjectives from statements about the properties of cognitive indifference relations associated with these predicates. I therefore conclude that systems such as DelTCS give us valuable insight into the relationship between cognitive judgements of indifference and reasoning patterns with scalar terms.

Keywords: vagueness; trivalent logic; adjectives; scalarity; evaluativity

1. Introduction

The question of how to construct an appropriate formal semantics for English predicates like *tall* and *short* has been an important but outstanding research problem in the fields of linguistics and analytical philosophy. It is well known that the logical analysis of such adjectives (known in the linguistics literature as *gradable* predicates) within systems that share the characterising properties of classical predicate logic is challenged by some of their empirical features. Firstly, gradable adjectives such as *tall* are *context-sensitive*; that is, different individuals can appear in the extension of *tall* in different contexts. For example, a man can be considered tall compared with a particular set of jockeys, but the same man might also not be considered tall within a set of basketball players. Thus, unlike in classical logic, where the extension of a predicate is fixed, the interpretation of gradable adjectives must be variable and relative to objects such as comparison classes. Secondly, once we fix the context of utterance (suppose to one in which we are evaluating the height of North American males), we can see a second challenging property of *tall*: it is *tolerant*; that is, its application is insensitive to small changes along a scalar dimension (Wright, 1975). For example, consider the relation '± one centimetre' (in the context described above). It seems absurd to think that changing someone's height by this much (in the context) could ever affect whether or not we would call them *tall*. In other words, (1) appears to be true.

*Email: heather.susan.burnett@gmail.com

(1) For all a_1, a_2, if a_1 is tall and a_1's and a_2's heights differ by at most one centimetre, then a_2 is also tall.

We call relations like '$\pm$ one centimetre' (in this context) *tolerance* relations or *indifference* relations, since they encode amounts of change that do not make a difference to categorisation. More abstractly, we will use $\sim_P$ to notate an indifference relation for a gradable predicate P, and then we observe that gradable predicates can (depending on context) satisfy the formula of (slightly extended) first order predicate logic in (2), commonly known as the *principle of tolerance*.

(2) **The principle of tolerance:**
$\forall x_1 \forall x_2 (P(x_1) \wedge x_1 \sim_P x_2 \rightarrow P(x_2))$.

The fact that gradable adjectives are tolerant has two important consequences for their semantic analysis: firstly, predicates that satisfy (2) appear to have *fuzzy boundaries*: suppose we take someone who is 1.9 metres tall, and suppose that we agree that, because we are talking about average male heights, he is tall. Furthermore, suppose that we have a long line of people ordered based on height and that their heights differ by only one centimetre each. The 1.9m tall man is at the front of the line, and there is someone who is only 1.5m tall at the end. We can agree that the last person is not tall. Given this set-up, there must be some point in this line at which we move from a tall person to a person who is not tall, but who is also only one centimetre shorter than the adjacent person. But since we agreed that *tall* satisfies (2), how can we determine this point? In other words, once the comparison class gets large enough, despite being competent users of the word *tall*, we are unable to identify a non-arbitrary boundary between this predicate and its negation. Secondly, predicates that are tolerant give rise to paradoxes for first order predicate logic and related systems known as *sorites* arguments. Such paradoxical arguments have the form in (3), where a_1 is understood to denote a clear case of the predicate P, a_k denotes a clear case of its negation, and there exists in the model a series of individuals that are all related by $\sim_P$ that starts with a_1 and ends with a_k.

(3) **The sorites paradox**

a. **Clear case**: $P(a_1)$.
b. **Clear non-case**: $\neg P(a_k)$.
c. **Sorites series**: $\forall i \in [1, n](a_i \sim_P a_{i+1})$.
d. **Tolerance**: $\forall x_1 \forall x_2((P(x_1) \wedge x_1 \sim_P x_2) \rightarrow P(x_2))$.
e. **Conclusion**: $P(a_k) \wedge \neg P(a_k)$.

Although taken individually they seem reasonable, as shown by (3-e), taken together the premises in (3) are inconsistent. Predicates that give rise to Soritical arguments are generally known as *vague* predicates in the literature and, therefore, based on their tolerant nature, vague predicates like *tall* challenge the very foundations of the kinds of semantic theories that linguists and philosophers often propose for the purpose of analysing natural language sentences.

In light of the variable and perhaps even incoherent nature of the semantic denotations of the *positive forms* of gradable adjectives (i.e., the form of the adjective in (4); Sapir, 1944), it comes as somewhat of a surprise to observe that adjectives such as *tall* and *short* can appear in (certain) clearly valid arguments that combine statements using their positive forms with statements using their *comparative* forms (i.e., the form of the adjective in (5)).

(4) **Statements using the positive form:**

a. John is *tall*.
b. Mary is *short*.

(5) **Statements using the comparative form:**

a. John is *taller* than Phil.
b. Mary is *shorter* than Sue.

A first example of such an argument is shown in (6): no matter how tallness is calculated and no matter what counts as being tall in the context, if John is tall and Mary is taller, then we know immediately that Mary will also be considered tall in the context.

(6) John is *tall* and Mary is *taller* than John. $\therefore$, Mary is *tall*.

The statements such as (6) that are analysed in this paper could all be, in principle, classified as what Fine (1975) calls *penumbral truths*; that is, they express logical relationships between sentences that hold even though the truth values of the sentences themselves maybe indefinite. The pattern in (6) is by no means limited to *tall* and can be recreated with the entire set of gradable adjectives; that is, any adjectives that can felicitously appear in a comparative construction (*short*, *expensive*, *friendly*, *wet*, *dirty*, etc.). In other words, if a predicate P has a comparative form,[1] we can observe that it satisfies the proposition in (7), where we notate the comparative relation associated with P as $>_P$.

(7) **Scalarity**:
$\forall x_1 \forall x_2 (P(x_1) \wedge x_2 >_P x_1 \rightarrow P(x_2))$.

In summary, although the ‘fuzzy’ nature of the positive forms of gradable predicates creates a challenge for their formal semantic analysis, this fuzziness hides a rich, logical structure that reveals itself in (among other places) arguments involving comparative constructions.

The goal of this paper is to illustrate how penumbral connections between statements involving the positive and comparative forms of different kinds of vague predicates can be captured using the Delineation Tolerant, Classical, Strict (DelTCS; Burnett, 2012b, 2013) non-classical logical framework. This framework has been used by Burnett (2012b) to capture relationships between the phenomena of context-sensitivity, vagueness and the properties of the abstract orderings (called *scales* in the linguistics literature) that are associated with different kinds of adjectival predicates, and it has also been used by Burnett (2014) to solve a set of paradoxes associated with the meanings of a certain subset of scalar predicates for delineation semantic frameworks (Klein, 1980, among others). This article presents yet another set of applications of this system: modelling reasoning patterns from the comparative to the positive with vague predicates in the adjectival domain.

Concretely, in this paper, I survey a number of classes of valid arguments that can be formed with different classes of gradable predicates in comparative constructions. We have already seen one class of such arguments: the *scalarity* arguments, exemplified in (7). I suggested that this argument held for all members of scalar adjectives. A second class of arguments that are of interest to us are those that involve *evaluativity* (Rett, 2008). The members of this class of arguments have a very simple form: they have a comparative construction as a premise and a positive construction as a conclusion. Crucially, not all scalar adjectives are evaluative. For example, the argument in (8) that uses the predicate *tall* is not valid: Mary can be taller than John, but yet still not be considered tall in the context.

(8) Mary is taller than John $\not\models$ Mary is tall.

However, it is proposed by Unger (1975), Lewis (1979), Cruse (1986), Yoon (1996), and Rett (2008), amongst others, that the same argument is valid with an adjective such as *wet* that is a member of a different semantic class (to be defined below) (9).

(9) This towel is wetter than that towel $\models$ This towel is wet.

Thus, we see variation in the penumbral connections that exist between comparative and positive forms of different classes of adjectives. The main goal of this paper, then, is to show how the observed variation in the reasoning patterns of vague predicates can be captured within a multi-valued non-classical logical framework such as DelTCS.

The paper is organised as follows. In section 2, I present the basic inference patterns associated with scalar adjectives. In particular, we will see arguments in favour of distinguishing between three sub-classes of these adjectives based on their inferential properties:

(10) a. **Relative adjectives**: *tall*, *short*, *wide*, *narrow*, *intelligent*, *friendly*, etc.
b. **Total adjectives**: *dry*, *clean*, *straight*, *empty*, *full*, etc.
c. **Partial adjectives**: *wet*, *dirty*, *bent*, *sick*, *awake*, etc.

Then, in section 3, I outline the DelTCS system and give an analysis of the semantics and pragmatics of gradable adjectives in it. In section 4, I show that the scalarity and evaluativity arguments presented in section 2 are valid in this system. Finally, section 5 gives a summary of the results of the approach described in this work and concludes.

2. Scalarity, maximality and evaluativity

As discussed by many authors (such as Sapir, 1944; Unger, 1975; Lewis, 1979; Cruse, 1986; Yoon, 1996; Kennedy, 2007; Rett, 2008), the relative/total/partial three-way distinction has consequences for the kinds of arguments that can be formed with the comparative and positive forms of the respective adjectives.[2] In this section, I give an overview of both the similarities and the differences in reasoning patterns with these three classes of adjectives.

2.1. *Scalarity*

We first observe that members of all three scale structure classes validate the *scalarity* argument that was discussed in the introduction and is schematised in (11).

(11) **Scalarity**:
$\forall x_1 \forall x_2 (P(x_1) \wedge x_2 >_P x_1 \rightarrow P(x_2))$.

For example, although the criteria of application of relative predicates like *tall* and *short* are highly variable, the arguments in (12) are clearly valid.[3]

(12) a. Mary is taller than John and John is tall. $\therefore$ Mary is tall.
b. Mary is shorter than John and John is short. $\therefore$ Mary is short.

We find the same pattern with partial adjectives like *wet* and *dirty*.

(13) a. This towel is wetter than that towel and that towel is wet. $\therefore$ This towel is wet.
b. This room is dirtier than that room and that room is dirty. $\therefore$ This room is dirty.

Finally, as shown in (14), total predicates also conform to the schema in (11).

(14) a. This room is emptier than that room and that room is empty. ∴ This room is empty.
b. Stick A is straighter than stick B and stick B is straight. ∴ Stick A is straight.

2.2. Maximality

Although the arguments in (14) are surely valid (i.e., how could a room that is emptier than an empty room fail to be empty?), they are a little strange. In particular, many authors (such as Sapir, 1944; Unger, 1975; Lewis, 1979; Cruse, 1986; Kennedy & McNally, 2005) have expressed some unease with the conjunction of the premises in (14), namely the idea that some distinct object can be emptier/straighter than some object that is already empty/straight. Generally, these authors propose that total predicates pick out those objects that are at the top endpoint of the adjective's associated scale, i.e., those objects that are maximally/completely *straight*, *empty*, *flat*, etc. In other words, total adjectives are often proposed to satisfy the proposition in (15), and the arguments in (16) are proposed to be valid.

(15) **Maximality**:
$\forall x_1 \forall x_2 (x_1 >_Q x_2 \rightarrow \neg Q(x_2))$.

(16) a. This room is emptier than that room. ∴ That room is not empty.
b. Stick A is straighter than stick B. ∴ Stick B is not straight.

Thus, in this way of thinking about things, the scalarity argument (11) would be valid for total predicates, but only because its premises are contradictory (i.e., it would be vacuously true).

But is it really impossible for something to be emptier/straighter/cleaner than something that is considered empty/straight/clean (in the context of utterance)? In fact, the validity of maximality arguments with total predicates is a subject of disagreement in the literature. For example, Kennedy and McNally (2005) report the argument in (17-a) as being a contradiction (in line with the validity of (16)), whereas Rotstein and Winter (2004) claim that the almost identical sentence (17-b) can be true.[4]

(17) a. # The red towel is cleaner than the blue one, but both are clean (Kennedy & McNally, 2005).
b. Both towels are clean but the red one is cleaner than the blue one (Rotstein & Winter, 2004).

What is going on in (17)? Are the differences in judgements between authors due to dialectal differences in the semantics assigned to *clean* or to something else? Following authors such as Lewis (1979) and Sassoon and Toledo (2011), I propose that the contradictory and non-contradictory readings of sentences like (17) are the result of differences associated with the level of precision with which the statements are evaluated. Particularly, I propose that the maximality proposition (15) is valid for (what I will call) the precise or *strict* interpretation of total predicates. So sentences like (17) are contradictions under this interpretation. However, (what I will call) the loose or *tolerant* interpretation of these predicates does not satisfy (15), and the sentences in (17) are not contradictions under this interpretation.

Furthermore, I note that the strict and tolerant interpretations of total predicates can be disambiguated through a variety of linguistic means. For example, the strict interpretation of a total adjective can be targeted through the use of adverbials like *absolutely/perfectly* (18-a), through the use of emphatic focus ((18-b); cf. Kennedy, 2007; Unger, 1975), and through the use of the *contrastive focus reduplication* construction ((18-c); Ghomeshi, Jackendoff, Rosen & Russell, 2004).

(18) **Strict interpretation**:

a. Stick A is straighter than stick B. $\models$ Stick B is not *absolutely/perfectly* straight.
b. Stick A is straighter than stick B. $\models$ Stick B is not STRAIGHT.
c. Stick A is straighter than stick B. $\models$ Stick B is not STRAIGHT straight.

The tolerant interpretation of total predicates can be targeted by 'hedges' like *loosely speaking*, *approximately* or *roughly* (Lakoff, 1973). As shown in (19), when the adjective is modified by a hedge that forces its tolerant interpretation, the maximality argument no longer goes through. The appropriate counter-example involves the case where stick A is perfectly straight and stick B has some small bends in it (so is still, loosely speaking, straight).

(19) **Tolerant interpretation**:

a. Stick A is straighter than stick B. $\not\models$ Stick B is not *loosely speaking* straight.
b. Stick A is straighter than stick B. $\not\models$ Stick B is not *approximately* straight.
c. Stick A is straighter than stick B. $\not\models$ Stick B is not *roughly* straight.

In summary, we have seen that it is important to make a distinction between the interpretation of a predicate under its strict use and its interpretation under its tolerant use. In particular, I argued that the maximality argument is valid only for the strict interpretation of the predicate. In what follows, in order to be fully explicit about the arguments associated with the different kinds of predicates, I will adopt a notation that makes reference to the different levels of precision. In particular, I will write $[\![\phi]\!]^s$ to refer to the strict interpretation of an expression ϕ, and I will write $[\![\phi]\!]^t$ to signify its tolerant interpretation. In section 3, I will make a proposal (based on previous work such as Cobreros et al., 2012) about how strict and tolerant interpretations are calculated.

Coming back to the discussion of the data: I argued that the strict interpretation of the maximality proposition ought to be valid for a total predicate Q in our logic (20), but that the tolerant interpretation of this proposition ought not to be valid (21).

(20) For all models M,
$[\![\forall x_1 \forall x_2 (x_1 >_Q x_2 \rightarrow \neg Q(x_2))]\!]^s_M = 1.$

(21) *False:* For all models M,
$[\![\forall x_1 \forall x_2 (x_1 >_Q x_2 \rightarrow \neg Q(x_2))]\!]^t_M = 1.$

Furthermore, since the scalarity proposition holds regardless of whether we interpret the positive form of the adjective strictly or loosely, the positions in (22) should be valid (for all kinds of gradable adjectives).

(22) For all models M and all predicates P,

a. $[\![\forall x_1 \forall x_2 (P(x_1) \wedge x_2 >_P x_1 \rightarrow P(x_2))]\!]^s_M = 1.$
b. $[\![\forall x_1 \forall x_2 (P(x_1) \wedge x_2 >_P x_1 \rightarrow P(x_2))]\!]^t_M = 1.$

Finally, if we consider both relative adjectives and partial adjectives, we see that they show a different logical pattern from total adjectives: no matter whether these predicates are interpreted strictly or tolerantly,[5] they never satisfy *maximality*.

(23) a. John is taller than Mary $\not\models$ Mary is not tall.
b. John is shorter than Mary $\not\models$ Mary is not short.

(24) a. This towel is wetter than that towel $\not\models$ This towel is not wet.
b. This table is dirtier than that table $\not\models$ That table is not dirty.

In other words, even if we evaluate *tall/wet* strictly ($\approx$ 'really tall/wet') (25) or tolerantly ($\approx$ 'borderline tall/wet') (26), we cannot conclude the negation of the positive form from the comparative construction.

(25) **Strict interpretation**:

a. John is taller than Mary $\not\models$ Mary is not TALL tall.
b. This towel is wetter than that towel $\not\models$ That towel is not WET wet.

(26) **Tolerant interpretation**:

a. John is taller than Mary $\not\models$ Mary is not *roughly* tall.
b. This towel is wetter than that towel $\not\models$ That towel is not (at least) *strictly speaking* wet.

I therefore conclude that while the *scalarity* proposition is valid for all adjectives under all interpretations, only total adjectives under a strict interpretation satisfy *maximality*.

2.3. *Evaluativity*

However, partial adjectives (*wet*, *dirty*, *bent*, etc.) can be used in some arguments that are not valid with relative and total adjectives. In particular, as observed by Yoon (1996), Rotstein and Winter (2004), Kennedy (2007) and Rett (2008), comparatives formed with partial predicates are *evaluative*, i.e., we can reason from the comparative form to the positive form in a way such as (27).

(27) a. This towel is wetter than that towel $\therefore$ This towel is wet.
b. Your dress is dirtier than my dress $\therefore$ Your dress is dirty.

I will refer to this argument as the *evaluativity* proposition in (28).

(28) **Evaluativity**:
$\forall x_1 \forall x_2 (x_1 >_P x_2 \rightarrow P(x_1))$.

Again, we must be cautious to observe under which interpretation(s) of the positive form of the adjective the evaluativity argument goes through. Particularly, as shown in (29), it does not go through if the partial predicate is interpreted strictly: this towel can be wetter than that towel, but still be almost dry, i.e., not clearly wet.

(29) **Strict interpretation**:

a. This towel is wetter than that towel $\not\models$ This towel is WET wet.
b. Your dress is dirtier than my dress $\not\models$ Your dress is DIRTY dirty.

Instead, if we know that this towel is wetter than that towel, we know that this towel must be at least tolerantly wet. I highlight here that it is important not to confuse the English expression *strictly speaking* with the metalinguistic term *strictly* that we have been using to refer to the definite/precise interpretation of the predicate. They look very similar, but have very different meanings: *strictly speaking wet* → *not very wet*, while *strictly wet* → *very wet*. I apologise if this causes confusion.

(30) This towel is wetter than that one. ∴ This towel is (at least) *strictly speaking* wet.

Thus, in our logic, we would like for a partial predicate R to satisfy *evaluativity* on its tolerant interpretation:

(31) For all models M,
$[\![\forall x_1 \forall x_2 (x_1 >_R x_2 \rightarrow R(x_1))]\!]^t_M = 1.$

(32) It is not the case that, for all models M,
$[\![\forall x_1 \forall x_2 (x_1 >_R x_2 \rightarrow R(x_1))]\!]^s_M = 1.$

If we now consider relative adjectives, we see a different pattern: even if we are being as tolerant as possible, we cannot conclude that John is tall (in the context) from knowing that he is taller than Mary. So *evaluativity* should be invalid for relative predicates (34).

(33) John is taller than Mary $\not\models$ John is tall.
(34) It is not the case that for all models M, relative predicates P and $m \in \{s, t\}$,
$[\![\forall x_1 \forall x_2 (x_1 >_P x_2 \rightarrow P(x_1))]\!]^m_M = 1.$

Finally, at first glance, total adjectives show the same behaviour: even if the positive form of the adjective is interpreted tolerantly, the arguments in (35) are invalid.

(35) a. This room is emptier than that room $\not\models$ This room is (loosely speaking) empty.
b. Stick A is straighter than stick B $\not\models$ Stick B is (loosely speaking) straight.

(36) For all models M, total predicates Q and $m \in \{s, t\}$,
$[\![\forall x_1 \forall x_2 (x_1 >_Q x_2 \rightarrow Q(x_1))]\!]^m_M = 1.$

This being said, I highlight that there is still a difference between the behaviour of total and relative adjectives in the *evaluativity* argument. Although adjectives like *empty* and *straight* do not satisfy (28) as such, as observed by Burnett (2012b), subjects of comparative statements with total adjectives are required to be at least somewhat close to tolerantly satisfying the predicate. For example, (37-a) is very strange if both rooms are almost full even if this room has slightly fewer objects than that room. Likewise, (37-b) is bizarre if both sticks are very curvy, even if stick A is less so.

(37) a. This room is emptier than that room.
b. Stick A is straighter than stick B.

Relative predicates do not have this property: John can be considered quite short in the context and (38) is still fine.

(38) John is taller than Mary.

We will see that the logical analysis presented in the next section will provide an account of this 'pseudo-evaluativity' property that we see with total absolute adjectives.

2.4. Summary

In this section, I argued that in order to arrive at an appropriate description of the data, it is important to pay close attention to the ways in which the predicates are interpreted and, furthermore, I showed how the insertion of modifiers like *loosely speaking*, *strictly*

Table 1. Reasoning patterns associated with relative, partial and total adjectives.

ARGUMENT	RELATIVE		TOTAL		PARTIAL	
	$[\![\cdot]\!]^t$	$[\![\cdot]\!]^s$	$[\![\cdot]\!]^t$	$[\![\cdot]\!]^s$	$[\![\cdot]\!]^t$	$[\![\cdot]\!]^s$
Scalarity	✓	✓	✓	✓	✓	✓
Maximality	×	×	×	✓	×	×
Evaluativity	×	×	×	×	✓	×

speaking, and contrastive focus reduplication (i.e., TALL *tall*) can linguistically distinguish these interpretations. Once questions of precision are clarified, I showed that that the three classes of gradable predicates can be used in different kinds of arguments involving their comparative and positive forms. A summary of the reasoning patterns discussed in this paper is shown in Table 1.

In the next section, I will present a new logical framework that can model these patterns, and in section 4, I will show how the reasoning patterns displayed in Table 1 are theorems in the *DelTCS* system.

3. Delineation tolerant, classical, strict (DelTCS)

I start by defining the language of *DelTCS* and then I give its semantics.

3.1. The language

The vocabulary consists of the following expressions:

(1) A series of individual constants: $a_1, a_2, a_3 \ldots$
(2) A series of individual variables: $x_1, x_2, x_3 \ldots$
(3) Three series of unary predicate symbols:

- Relative scalar adjectives: P, P_1, P_2, $P_3 \ldots$
- Total absolute scalar adjectives: Q, Q_1, Q_2, $Q_3 \ldots$
- Partial absolute scalar adjectives: R, R_1, R_2, $R_3 \ldots$

(4) For every unary predicate symbol P, there is a binary predicate $>_P$.
(5) For every unary predicate symbol P, there is a binary predicate $\sim_P$.
(6) Quantifier $\forall$ and connectives $\wedge$, $\vee$, $\neg$ and $\rightarrow$, plus parentheses.

I highlight that the vocabulary of *DelTCS* differs from that of first order predicate logic in that 1) we have three classes of predicates (relative, total and partial); 2) we have a set of distinguished binary predicates that notate the comparative relations (the $>_P$s); and 3) we have a set of distinguished binary predicates that notate the indifference relations (the $\sim_P$s). The syntax of *DelTCS* is as follows:

(1) Variables and constants (and nothing else) are terms.
(2) If t is a term and P is a predicate symbol, then $P(t)$ is a well-formed formula (wff).
(3) If t_1 and t_2 are terms and P is a predicate symbol, then $t_1 >_P t_2$ is a wff.
(4) If t_1 and t_2 are terms and P is a predicate symbol, then $t_1 \sim_P t_2$ is a wff.
(5) For any variable x, if ϕ and ψ are wffs, then $\neg\phi$, $\phi \wedge \psi$, $\phi \vee \psi$, $\phi \rightarrow \psi$, and $\forall x\phi$ are wffs.
(6) Nothing else is a wff.

3.2. The semantics

We now turn to the heart of the proposal: the semantics of *DelTCS*. *Delineation TCS* is so-called because it is a combination of two existing logical systems: delineation semantics and TCS. More specifically, I combine (a simplified version) of Klein's (1980) comparison-class-based *delineation semantics* system for modelling the relationship between context-sensitivity and gradability with the non-classical *Tolerant*, *Classical*, *Strict* system of Cobreros et al. (2012) for modelling the relationship between indifference relations and vagueness. In this framework (as in Klein's system), scalar adjectives denote sets of individuals and, furthermore, they are evaluated with respect to comparison classes, i.e., subsets of the domain D. The basic idea is that the extension of a gradable predicate can change depending on the set of individuals that it is being compared with. In other words, the semantic denotation of the positive form of the scalar predicate (i.e., *tall*) can be assigned a different set of individuals in different comparison classes.

We therefore start with a basic model (called a *C(lassical) model* in the terminology of Cobreros et al., 2012) in which the interpretation of predicates is relativised to comparison classes as follows:

Definition 1. ***C-model***. *A c-model is a tuple $M = \langle D, m\rangle$ where D is a non-empty domain of individuals, and m is a function from pairs consisting of a member of the non-logical vocabulary and a comparison class (a subset of the domain) satisfying:*

- *For each individual constant a_1, $m(a_1) \in D$.*
- *For each $X \subseteq D$ and for each predicate P, $m(P, X) \subseteq X$.*

We now introduce the additional structure that forms the contribution of the TCS system (Cobreros et al., 2012) for modelling the properties of vague language. This system was developed as a way to preserve the intuition that vague and imprecise predicates[6] are *tolerant* (i.e., satisfy $\forall x\forall y[P(x)\ \&\ x \sim_P y \rightarrow P(y)]$, where $\sim_P$ is a 'little by little' or indifference relation for a predicate P) without running into the Sorites paradox. Cobreros et al. (2012) adopt a non-classical logical framework with three notions of satisfaction: classical satisfaction, tolerant satisfaction, and its dual, strict satisfaction. Formulas are tolerantly/strictly satisfied based on classical truth and predicate-relative, possibly non-transitive *indifference relations*. In order to incorporate the additional structure of TCS into our delineation system, we extend the c-model (defined in Definition 1) into a *T(olerant) model* by adding indifference relations ($\sim_P$s)[7] into the model structure as follows:

Definition 2. ***T-model***. *A t-model is a tuple $M = \langle D, m, \sim\rangle$, where $\langle D, m\rangle$ is a c-model and $\sim$ is a function from predicate/comparison class pairs such that:*

- *For all P and all $X \subseteq D$, $\sim_P^X$ is a binary relation on X.*

Note that I will use $\sim_P^X$ to notate an indifference relation in a model at a comparison class X; whereas, the comparison classes are not syntactically represented in the language. For the moment, we will allow the $\sim_P^X$s to be any binary relation on the members of X, but in the next subsection, I will discuss what appropriate constraints should be put on the indifference relations.

The interpretation of variables is done in the standard way (on assignment):

Definition 3. ***Assignment***. *An assignment for a c/t-model M is a function $g : \{x_n : n \in \mathbb{N}\} \rightarrow D$ (from the set of variables to the domain D).*

Finally, formulas are interpreted in three ways: classically, tolerantly and strictly.

3.2.1. Classical interpretation

The classical interpretation of terms is given as follows:

Definition 4. ***Interpretation of terms*** *($[\![\cdot]\!]_{M,g}$). For a model M and an assignment g,*

(1) If x_1 is a variable, $[\![x_1]\!]_{M,g} = g(x_1)$.
(2) If a_1 is a constant, $[\![a_1]\!]_{M,g} = m(a_1)$.

In what follows, for an interpretation $[\![\cdot]\!]_{M,g}$, a variable x_1, and a constant a_1, let $g[a_1/x_1]$ be the assignment for M which maps x_1 to a_1 but agrees with g on all variables that are distinct from x_1.

The definition of classical satisfaction is given in Definition 5 below. Since the interpretations of predicates and therefore of atomic formulas are relativised to comparison classes (CCs), I propose that atomic formulas will be true if the extension of the argument is included in the extension of the predicate in the CC; they will be false if the argument is in the complement of the predicate in the CC, and they will be undefined otherwise. This proposal aims to account for the strangeness of cases such as (39).

(39) # Mary is tall for a man.

In Definition 5, formulas are interpreted relative to a model, an assignment function, and a comparison class; however, in what follows, for readability considerations, I will often omit the model and assignment notation, writing only $[\![\cdot]\!]_X$ for $[\![\cdot]\!]_{M,g,x}$.

Definition 5. ***Classical satisfaction*** *($[\![\cdot]\!]$). For all interpretations $[\![\cdot]\!]_{M,g}$, all $X \in \mathcal{P}(D)$, all formulas ϕ, ψ, all predicates P, and all terms t_1, t_2,*

(1) $$[\![P(t_1)]\!]_{M,g,X} = \begin{cases} 1 & \text{if } [\![t_1]\!]_{M,g} \in m(P,X) \\ 0 & \text{if } [\![t_1]\!]_{M,g} \in X - m(P,X) \\ i & \text{otherwise} \end{cases}$$

(2) $$[\![t_1 >_P t_2]\!]_{M,g,X} = \begin{cases} 1 & \text{if there is some } X' \subseteq D : [\![P(t_1)]\!]_{M,g,X'} = 1 \\ & \quad \text{and } [\![P(t_2)]\!]_{M,g,X'} = 0 \\ 0 & \text{otherwise} \end{cases}$$

(3) $$[\![t_1 \sim_P t_2]\!]_{M,g,X} = \begin{cases} 1 & \text{if } t_1 \sim_P^X t_2 \\ 0 & \text{otherwise} \end{cases}$$

(4) $$[\![\neg\phi]\!]_{M,g,X} = \begin{cases} 1 & \text{if } [\![\phi]\!]_{M,g,X} = 0 \\ 0 & \text{if } [\![\phi]\!]_{M,g,X} = 1 \\ i & \text{otherwise} \end{cases}$$

(5) $$[\![\phi \wedge \psi]\!]_{M,g,X} = \begin{cases} 1 & \text{if } [\![\phi]\!]_{M,g,X} = 1 \text{ and } [\![\psi]\!]_{M,g,X} = 1 \\ 0 & \text{if } \{[\![\phi]\!]_{M,g,X}, [\![\psi]\!]_{M,g,X}\} = \{1,0\} \text{ or } \{0\} \\ i & \text{otherwise} \end{cases}$$

(6) $$[\![\phi \vee \psi]\!]_{M,g,X} = \begin{cases} 1 & \text{if } [\![\phi]\!]_{M,g,X} = 1 \text{ or } [\![\psi]\!]_{M,g,X} = 1 \\ 0 & \text{if } [\![\phi]\!]_{M,g,X} = [\![\psi]\!]_{M,g,X} = 0 \\ i & \text{otherwise} \end{cases}$$

(7) $$[\![\phi \rightarrow \psi]\!]_{M,g,X} = \begin{cases} 1 & \text{if } [\![\phi]\!]_{M,g,X} = 0 \text{ or } [\![\psi]\!]_{M,g,X} = 1 \\ 0 & \text{if } [\![\phi]\!]_{M,g,X} = 1 \text{ and } [\![\psi]\!]_{M,g,X} = 0 \\ i & \text{otherwise} \end{cases}$$

$$(8)\ [\![\forall x_1\phi]\!]_{M,g,X} = \begin{cases} 1 & \textit{if for every } a_1 \in X, [\![\phi]\!]_{M,g[a_1/x_1],X} = 1 \\ 0 & \textit{if for some } a_1 \in X, [\![\phi]\!]_{M,g[a_1/x_1],X} = 0 \\ i & \textit{otherwise} \end{cases}$$

The interpretation of the comparative predicates ($>_P$s) is taken directly from Klein (1980): the delineation semantics framework takes the positive form of the adjective as basic and derives the semantics of the comparative form from quantification over comparison classes. Informally, *John is taller than Mary* is true in the case that there is some comparison class with respect to which John counts as tall and Mary counts as not tall. This being said, as it stands, the analysis of the comparative in Definition 5 is very weak and its interpretation needs to be constrained to rule out non-comparative-like relations from being instantiated by the $>_P$s.[8] In this work, I will adopt the set of constraints on the application of gradable predicates presented in van Benthem (1982, 1990) (although see van Rooij, 2011a for another set of constraints that could also be adopted in this framework). In his studies, van Benthem proposes three axioms governing the behaviour of individuals across comparison classes. They are the following (presented in my notation):

For all predicates P_1, all interpretations $[\![\cdot]\!]_{M,g}$, all $X \subseteq D$ and $a_1, a_2 \in X$ such that $[\![P_1(a_1)]\!]_{M,g,X} = 1$ and $[\![P_1(a_2)]\!]_{M,g,X} = 0$,

(1) **No reversal (NR)**: There is no $X' \subseteq D$ such that $[\![P_1(a_2)]\!]_{M,g,X'} = 1$ and $[\![P(a_1)]\!]_{M,g,X'} = 0$.
(2) **Upward difference (UD)**: For all $X' \subseteq D$, if $X \subseteq X'$, then there are some a_3, a_4 : $[\![P_1(a_3)]\!]_{M,g,X'} = 1$ and $[\![P_1(a_4)]\!]_{M,g,X'} = 0$.
(3) **Downward difference (DD)**: For all $X' \subseteq D$, if $X' \subseteq X$ and $a_1, a_2 \in X'$, then there are some a_3, a_4 : $[\![P_1(a_3)]\!]_{M,g,X'} = 1$ and $[\![P_1(a_4)]\!]_{M,g,X'} = 0$.

No reversal states that if $x >_P y$, there is no X' such that y is in the extension of P in X', but x is not. *Upward difference* states that if, in the comparison class X, there is a P/*not* P contrast, then a P/*not* P contrast is preserved in every larger CC. Finally, *downward difference* says that if in some comparison class X there is a *P/not P* contrast involving x and y, then there remains a contrast in every smaller CC that contains both x and y. In his studies, van Benthem shows that these axioms give rise to *strict weak orders*: irreflexive, transitive and almost connected relations.[9]

Definition 6. ***Strict weak order***. *A relation $>$ is a strict weak order only in the case that $>$ is* irreflexive, transitive, *and* almost connected.

As discussed in Klein (1980), van Benthem (1990) and van Rooij (2011b), strict weak orders (also known as *ordinal scales* in measurement theory) intuitively correspond to the types of relations expressed by many kinds of comparative constructions.[10] Thus, Theorem 7 shows that orders lexicalised by the comparative form of gradable predicates can be constructed from the context-sensitivity of the positive form and certain axioms governing the application of the predicate across different contexts.

Theorem 7. ***Strict weak order***. *For all P, $>_P$ is a strict weak order.*

Proof. This can be found in van Benthem (1982); van Benthem (1990, p. 116). ■

Klein's system and van Benthem's enrichments of it were originally designed to model the extreme context-sensitivity of relative adjectives; however, there are reasons to think that total and partial adjectives do not have the same context-sensitivity properties. For example, as observed by Kyburg and Morreau (2000), Kennedy (2007) and Syrett, Kennedy and Lidz

(2010), among others, adjectives like *tall/short* and *empty/straight* differ in whether they can 'shift' their thresholds (i.e., criteria of application) to distinguish between two individuals in a two-element comparison class when they appear in a definite description. For example, suppose there are two containers (A and B), and neither of them are particularly tall/short; however, A is (noticeably) taller/shorter than B. In this situation, if someone asks me (40-a), then it is very clear that I should pass A. Now suppose that container A has less liquid than container B, but neither container is particularly close to being completely empty. In this situation, unlike what we saw with *tall* and *short*, (40-b) is infelicitous. Similarly, if I have two very bent sticks, (40-c) is infelicitous, even if one is slightly less bent than the other.

(40) a. Pass me *the tall/short one*.
b. Pass me *the empty one*.
c. Pass me *the straight one*.

Partial predicates like *wet* and *dirty* display a similar pattern: if I have two towels, both of which are wet/dirty, even if one is wetter/dirtier than the other, the command in (41) is still not appropriate.

(41) Pass me *the wet/dirty one*.

In other words, the application of total and partial predicates across contexts is more restricted. Because of this feature, total and partial predicates are often called *absolute* adjectives, and an idea that has been present in the literature for a long time to account for the differences in context-sensitivity between relative and absolute adjectives[11] is that unlike *tall* and *long*, which have a context-sensitive meaning, adjectives like *straight*, *empty* and *wet* are not context-sensitive (hence the term *absolute* adjective). That is, in order to know what the straight sticks are or which rooms are empty, we do not need to compare them to a certain group of other individuals – we just need to look at their properties. To incorporate this idea into the delineation approach, I propose (following an idea from Burnett, 2012b; van Rooij, 2011b) that in a semantic framework based on comparison classes, what it means to be non-context-sensitive is to have your denotation be invariant across classes. Thus, for an absolute adjective Q and a comparison class X, it suffices to look at what the extension of Q is in the maximal CC, the domain D, in order to know what the extension of Q in X is. I therefore propose that a different axiom set governs the semantic interpretation of the members of the absolute class that does not apply to the relative class: the singleton set containing the *absolute adjective axiom*.

(42) **Absolute adjective axiom (AAA)**:
For all total and partial predicates Q_1, all interpretations $[\![\cdot]\!]_{M,g}$, all $X \in \mathcal{P}(D)$ and $a_1 \in X$,

(1) If $[\![Q_1(a_1)]\!]_{M,g,X} = 1$, then $[\![Q_1(a_1)]\!]_{M,g,D} = 1$.
(2) If $[\![Q_1(a_1)]\!]_{M,g,D} = 1$, and $[\![Q_1(a_1)]\!]_{M,g,X} \neq i$, then $[\![Q_1(a_1)]\!]_{M,g,X} = 1$.

In other words, the semantic denotation of an absolute adjective is set with respect to the total domain, and then, by the AAA, the interpretation of Q in D is replicated in each smaller comparison class. The replacement of NR, UD, and DD with the AAA has an important effect on the interpretation of the comparative relations associated with total and partial predicates. In particular, the relations denoted by the absolute and non-scalar comparative ($>_Q$) do not allow for the predicate to distinguish three distinct individuals. This fact is stated as Theorem 8:

Theorem 8. *If Q satisfies the AAA, then there is no model M such that, for distinct $x, y, z \in D$, $x >_Q y >_Q z$.*

Proof. Let Q satisfy the AAA. Suppose for a contradiction that there is some model M such that a_1, a_2, a_3 are distinct members of D, and $a_1 >_Q a_2 >_Q a_3$. Then, by Definition 5, there is some $X \subseteq D$ such that $[\![Q(a_1)]\!]_X = 1$ and $[\![Q(a_2)]\!]_X = 0$. Therefore, by the AAA, $[\![\mathbf{Q}(\mathbf{a_2})]\!]_\mathbf{D} = \mathbf{0}$. Furthermore, since $a_2 >_Q a_3$, there is some $X' \subseteq D$ such that $[\![Q(a_2)]\!]_{X'} = 1$ and $[\![Q(a_3)]\!]_{X'} = 0$. Since $[\![Q(a_2)]\!]_{X'} = 1$, by the AAA, $[\![\mathbf{Q}(\mathbf{a_2})]\!]_\mathbf{D} = \mathbf{1}$. $\perp$ So there is no model M such that, for distinct $a_1, a_2, a_3 \in D$, $a_1 >_Q a_2 >_Q a_3$. ■

So, even though the $>_Q$s are asymmetric, these relations establish an order only between individuals that are included in the classical extension of the predicate and those that are in the classical anti-extension. As we will see in section 4, this difference in the context-sensitivity of the semantic denotations of total and partial predicates will have important consequences for their associated reasoning patterns. I now turn to the definition of the tolerant and strict interpretations of formulas.

3.2.2. Tolerant/strict interpretations and vagueness

As in TCS, the tolerant and strict interpretations are interdefined based on the classical interpretation and the predicate-relative indifference relations, with $[\![\cdot]\!]^t$ and $[\![\cdot]\!]^s$ being duals.

Definition 9. ***Tolerant satisfaction*** *($[\![\cdot]\!]^t$). For all interpretations $[\![\cdot]\!]_{M,g}$, all $X \subseteq D$, all formulas ϕ, ψ, all predicates P, and all terms t_1, t_2,*

$$\text{(1)}\ [\![P(t_1)]\!]^t_{M,g,X} = \begin{cases} 1 & \textit{if there is some } a_1 \sim^X_P [\![t_1]\!]_{M,g} : [\![P(a_1)]\!]_{M,g,X} = 1 \\ 0 & \textit{if } [\![t_1]\!]_{M,g} \in X, \textit{and there is no } a_1 \in X : a_1 \sim^X_P [\![t_1]\!]_{M,g} \\ i & \textit{otherwise} \end{cases}$$

$$\text{(2)}\ [\![t_1 >_P t_2]\!]^t_{M,g,X} = \begin{cases} 1 & \textit{if there is some } X' \subseteq D : [\![P(t_1)]\!]^t_{M,g,X'} = 1 \\ & \quad \textit{and } [\![P(t_2)]\!]^t_{M,g,X'} = 0 \\ 0 & \textit{otherwise} \end{cases}$$

$$\text{(3)}\ [\![t_1 \sim_P t_2]\!]^t_{M,g,X} = \begin{cases} 1 & \textit{if } t_1 \sim^X_P t_2 \\ 0 & \textit{otherwise} \end{cases}$$

$$\text{(4)}\ [\![\neg\phi]\!]^t_{M,g,X} = \begin{cases} 1 & \textit{if } [\![\phi]\!]^s_{M,g,X} = 0 \\ 0 & \textit{if } [\![\phi]\!]^s_{M,g,X} = 1 \\ i & \textit{otherwise} \end{cases}$$

$$\text{(5)}\ [\![\phi \wedge \psi]\!]^t_{M,g,X} = \begin{cases} 1 & \textit{if } [\![\phi]\!]^t_{M,g,X} = 1 \textit{ and } [\![\psi]\!]^t_{M,g,X} = 1 \\ 0 & \textit{if } \{[\![\phi]\!]^t_{M,g,X}, [\![\psi]\!]^t_{M,g,X}\} = \{1,0\} \textit{ or } \{0\} \\ i & \textit{otherwise} \end{cases}$$

$$\text{(6)}\ [\![\phi \vee \psi]\!]^t_{M,g,X} = \begin{cases} 1 & \textit{if } [\![\phi]\!]^t_{M,g,X} = 1 \textit{ or } [\![\psi]\!]^t_{M,g,X} = 1 \\ 0 & \textit{if } [\![\phi]\!]^t_{M,g,X} = [\![\psi]\!]^t_{M,g,X} = 0 \\ i & \textit{otherwise} \end{cases}$$

$$\text{(7)}\ [\![\phi \rightarrow \psi]\!]^t_{M,g,X} = \begin{cases} 1 & \textit{if } [\![\phi]\!]^t_{M,g,X} = 0 \textit{ or } [\![\psi]\!]^t_{M,g,X} = 1 \\ 0 & \textit{if } [\![\phi]\!]^t_{M,g,X} = 1 \textit{ and } [\![\psi]\!]^t_{M,g,X} = 0 \\ i & \textit{otherwise} \end{cases}$$

$$(8)\ [\![\forall x_1\phi]\!]^t_{M,g,X} = \begin{cases} 1 & \textit{if for every } a_1 \in X,\ [\![\phi]\!]^t_{M,g[a_1/x_1],X} = 1 \\ 0 & \textit{if for some } a_1 \in X,\ [\![\phi]\!]^t_{M,g[a_1/x_1],X} = 0 \\ i & \textit{otherwise} \end{cases}$$

Definition 10. ***Strict satisfaction*** *($[\![]\!]^s$). For all interpretations $[\![\cdot]\!]_{M,g}$, all $X \subseteq D$, all formulas ϕ, ψ, all predicates P, and all terms t_1, t_2,*

$$(1)\ [\![P(t_1)]\!]^s_{M,g,X} = \begin{cases} 1 & \textit{if for all } a_1 \sim^X_P [\![t_1]\!]_{M,g} : [\![P(a_1)]\!]_{M,g,X} = 1 \\ 0 & \textit{if } [\![t_1]\!]_{M,g} \in X, \textit{ and there is no } a_1 \in X : a_1 \sim^X_P [\![t_1]\!]_{M,g} \\ i & \textit{otherwise} \end{cases}$$

$$(2)\ [\![t_1 >_P t_2]\!]^s_{M,g,X} = \begin{cases} 1 & \textit{if there is some } X' \subseteq D : [\![P(t_1)]\!]^s_{M,g,X'} = 1 \\ & \quad \textit{and } [\![P(t_2)]\!]^s_{M,g,X'} = 0 \\ 0 & \textit{otherwise} \end{cases}$$

$$(3)\ [\![t_1 \sim_P t_2]\!]^s_{M,g,X} = \begin{cases} 1 & \textit{if } t_1 \sim^X_P t_2 \\ 0 & \textit{otherwise} \end{cases}$$

$$(4)\ [\![\neg\phi]\!]^s_{M,g,X} = \begin{cases} 1 & \textit{if } [\![\phi]\!]^t_{M,g,X} = 0 \\ 0 & \textit{if } [\![\phi]\!]^t_{M,g,X} = 1 \\ i & \textit{otherwise} \end{cases}$$

$$(5)\ [\![\phi \wedge \psi]\!]^s_{M,g,X} = \begin{cases} 1 & \textit{if } [\![\phi]\!]^s_{M,g,X} = 1 \textit{ and } [\![\psi]\!]^s_{M,g,X} = 1 \\ 0 & \textit{if } \{[\![\phi]\!]^s_{M,g,X}, [\![\psi]\!]^s_{M,g,X}\} = \{1,0\} \textit{ or } \{0\} \\ i & \textit{otherwise} \end{cases}$$

$$(6)\ [\![\phi \vee \psi]\!]^s_{M,g,X} = \begin{cases} 1 & \textit{if } [\![\phi]\!]^s_{M,g,X} = 1 \textit{ or } [\![\psi]\!]^s_{M,g,X} = 1 \\ 0 & \textit{if } [\![\phi]\!]^s_{M,g,X} = [\![\psi]\!]^t_{M,g,X} = 0 \\ i & \textit{otherwise} \end{cases}$$

$$(7)\ [\![\phi \to \psi]\!]^s_{M,g,X} = \begin{cases} 1 & \textit{if } [\![\phi]\!]^s_{M,g,X} = 0 \textit{ or } [\![\psi]\!]^s_{M,g,X} = 1 \\ 0 & \textit{if } [\![\phi]\!]^s_{M,g,X} = 1 \textit{ and } [\![\psi]\!]^s_{M,g,X} = 0 \\ i & \textit{otherwise} \end{cases}$$

$$(8)\ [\![\forall x_1\phi]\!]^s_{M,g,X} = \begin{cases} 1 & \textit{if for every } a_1 \in X,\ [\![\phi]\!]^s_{M,g[a_1/x_1],X} = 1 \\ 0 & \textit{if for some } a_1 \in X,\ [\![\phi]\!]^s_{M,g[a_1/x_1],X} = 0 \\ i & \textit{otherwise} \end{cases}$$

Some comments concerning the definitions presented above are in order. Firstly, in this framework, an atomic formula $P(a_1)$ is tolerantly true relative to a comparison class in the case that *there is* some individual a_2 in the comparison class to which a_1 is indifferent, and the classical interpretation of the predicate applies to a_2. Thus, the tolerant extension of a predicate (i.e., $\{a_2 : [\![P(a_2)]\!]^t_X = 1\}$) contains those individuals that count as 'loosely' or 'roughly' P in the context (as well as those that are classically P). Correspondingly, an atomic formula $P(a_1)$ is strictly true relative to a comparison class in the case that *all* the individuals to which a_1 is indifferent satisfy the classical interpretation of the predicate. Thus, the strict extension of a predicate (i.e., $\{a_2 : [\![P(a_2)]\!]^s_X = 1\}$) contains just those individuals that count as 'definitely' or 'really' P in the context.

Secondly, I note that the tolerant/strict interpretations of the binary predicates $>_P$ and $\sim_P$ are defined in a parallel manner to their classical interpretations. In particular, following Cobreros et al. (2012), the indifference predicates ($\sim_P$s) are given a 'rigid' interpretation; that is, $[\![a_1 \sim_P a_2]\!]_X = [\![a_1 \sim_P a_2]\!]^t_X = [\![a_1 \sim_P a_2]\!]^s_X$. Furthermore, we will want to refer to the relations denoted by the tolerant and strict interpretations of the comparative predicates ($>_P$), so I will notate these relations $>^t_P$ and $>^s_P$ respectively.

Thirdly, we can observe that, by virtue of the existential quantifier in Definition 9, the tolerant denotation of a predicate will include its classical denotation, but may also include additional individuals in its anti-extension. Thus, the tolerant denotation of a predicate like *empty* might be the property of being (at least) 'loosely empty' or 'approximately empty'.

A final and vital precision must be made concerning the definition of the $\sim^X_P$ relations. These objects constitute the heart of the system since they form the basis of the calculation of tolerant and strict interpretations. However, at the moment, we have not yet made any proposals about what properties these relations might have. Like the case discussed in the previous subsection with the interpretations of predicates across comparison classes, if we do not say anything about how indifference relations can be established across comparison classes then the $\sim^X_P$s will not look at all like the cognitive indifference relations that they are supposed to be modelling. In what follows, I will propose a series of constraints that the $\sim$ function must satisfy across comparison classes. These constraints are meant to model cognitive indifference with respect to the *application* of the predicate (i.e., the categorisation process), rather than some kind of metaphysical or perceptual similarity relation. These axioms are also adopted in Burnett's (2012b) delineation analysis of adjectival scale structure, and they are discussed in greater detailed and exemplified in that paper.

The first property that is generally proposed to characterise indifference/similarity relations is reflexivity (cf., among many others, Luce, 1956; Pogonowski, 1981; Cobreros et al., 2012). Intuitively, every individual is indifferent from itself. Thus, we adopt the constraint in (43) that enforces reflexivity across CCs.

(43) **Reflexivity (R)**: For all predicates P, all interpretations $[\![\cdot]\!]_{M,g}$, all $X \subseteq D$, for all $a_1 \in X$, $a_1 \sim^X_{P_1} a_1$.

In addition to being reflexive, indifference and similarity relations are generally proposed to be symmetric (e.g., the original formulation of TCS in Cobreros et al., 2012). At first glance, this seems reasonable: if an individual a is considered indifferent from an individual b, then surely b must also be considered indifferent from a. Indeed, this seems appropriate for relative adjectives like *tall*: if John is tall and Phil is very slightly shorter than John, in most contexts, we will also apply *tall* to Phil. Likewise, if John is not tall and Phil is slightly taller than John, then surely we will still not apply *tall* to Phil. However, I argue, following Burnett (2012a,b), that there are reasons to think that symmetry of indifference relations is not appropriate for absolute predicates. In particular, limited non-symmetries in the application of total and partial predicates can be seen when it comes to relating members of an absolute predicate's classical extension with members of its anti-extension. For example, consider a total predicate like *straight*. Depending on the context, it is often possible to consider a stick that is very slightly bent to be *straight*; in other words, a stick that is not straight can sometimes be considered indifferent from a stick that is perfectly straight with respect to the application of the predicate. However, we can observe that, no matter what the context is, a perfectly straight stick can never be considered *not straight*, i.e., we should prohibit indifference relations that relate perfectly straight sticks to bent sticks. Likewise, for a partial predicate *wet*, it is sometimes possible to consider a towel with a few drops of water on it as *not wet*; however, the reverse is never possible. I therefore propose that the establishment indifference relations in the model are subject to different symmetry-related constraints for different classes of predicates. In particular, relative predicates will have symmetric $\sim_P$s, but the indifference relations associated with total and partial predicates will not be symmetric when they relate members of a predicate's classical extension with members of its anti-extension as follows:

(44) **(Non)Symmetry in $\sim$**:

(1) **Symmetry (S)**: For a relative predicate P_1, an interpretation $[\![\cdot]\!]_{M,g}$, and $a_1, a_2 \in D$, if $a_1 \sim^X_{P_1} a_2$, then $a_2 \sim^X_{P_1} a_1$.
(2) **Total axiom (TA)**: For a total predicate Q_1, an interpretation $[\![\cdot]\!]_{M,g}$, and $a_1, a_2 \in D$, if $[\![Q_1(a_1)]\!]_{M,g,D} = 1$ and $[\![Q_1(a_2)]\!]_{M,g,D} = 0$, then $a_2 \not\sim^X_{Q_1} a_1$, for all $X \subseteq D$.
(3) **Partial axiom (PA)**: For a partial predicate R_1, an interpretation $[\![\cdot]\!]_{M,g}$ and $a_1, a_2 \in D$, if $[\![R_1(a_1)]\!]_{M,g,D} = 1$ and $[\![R_1(a_2)]\!]_{M,g,D} = 0$, then $a_1 \not\sim^X_{R_1} a_2$, for all $X \subseteq D$.

The rest of the constraints on indifference relations across comparison classes that I will propose apply to all classes of predicates in the same way. The first general axiom that I propose is called *tolerant convexity*.[12]. *Tolerant convexity* says that if person A is indistinguishable from person B, and there is a person C lying in between persons A and B on the relevant tolerant scale, then A and C (the greater two of $\{A, B, C\}$) are also indistinguishable. The axiom is stated in (45).

(45) **Tolerant convexity**: For all predicates P_1, all interpretations $[\![\cdot]\!]_{M,g}$, all $X \subseteq D$, and all $a_1, a_2 \in X$,

- If $a_1 \sim^X_{P_1} a_2$ and there is some $a_3 \in X$ such that $a_1 \geq^t_{P_1} a_3 \geq^t_{P_1} a_2$, then $a_1 \sim^X_{P_1} a_3$.

I now propose a second axiom that is, in some sense, the dual of TC: *strict convexity* (46). This axiom says that if person A is indistinguishable from person B, and there is a person C lying in between persons A and B on the relevant strict scale, then B and C (the lesser two of $\{A, B, C\}$) are also indistinguishable.

(46) **Strict convexity**: For all predicates P_1, all interpretations $[\![\cdot]\!]_{M,g}$, all $X \subseteq D$, and all $a_1, a_2 \in X$,

- If $a_1 \sim^X_{P_1} a_2$ and there is some $a_3 \in X$ such that $a_1 \geq^s_{P_1} a_3 \geq^s_{P_1} a_2$, then $a_3 \sim^X_{P_1} a_2$.

The next constraint deals with how indifference relations can change across comparison classes. At the moment, $\sim_P$s can be established and destroyed in different comparison classes in a more or less arbitrary way, provided the previous three constraints are respected. But presumably we might want some more 'coherence' in the distribution of the $\sim_P$s. I therefore propose the following 'granularity' constraint:

(47) **Granularity (G)**: For all predicates P_1, all interpretations $[\![\cdot]\!]_{M,g}$, all $X \subseteq D$, and all $a_1, a_2 \in X$,

- If $a_1 \sim^X_{P_1} a_2$, then for all $X' \in \mathcal{P}(D) : X \subseteq X'$, $a_1 \sim^{X'}_{P_1} a_2$.

Granularity says that if person A and person B are indistinguishable in comparison class X, then they are indistinguishable in all supersets of X. This is meant to reflect the fact that the larger the domain is (i.e., the larger the comparison class is), the more things can cluster together. In other words, the larger the comparison class is, the more it is possible to collapse fine distinctions that were made in smaller comparison classes, and once you collapse such

Table 2. Constraints on the definition of $\sim$ for relative, total and partial predicates.

Constraint	Relative ($\sim_P$)	Total ($\sim_Q$)	Partial ($\sim_R$)
Reflexivity (R)	✓	✓	✓
Symmetry (S)	✓	×	×
Total axiom (TA)	×	✓	×
Partial axiom (PA)	×	×	✓
Tolerant convexity (TC)	✓	✓	✓
Strict convexity (SC)	✓	✓	✓
Granularity (G)	✓	✓	✓
Minimal difference (MD)	✓	✓	✓
Contrast preservation (CP)	✓	✓	✓

a ‘fine-grained’ distinction, you cannot make it again at a more ‘coarse-grained’ level It is loosely inspired by theories of granularity such as Hobbes (1985) in which distinctions made in a more complex theory are collapsed in simpler ‘coarse-grained’ theories (cf. also some similar remarks in van Rooij, 2011b).

While *granularity* talks about how indifference is preserved, the final two axioms (*minimal difference* and *contrast preservation*) deal with the preservation of differences across comparison classes. *Contrast preservation* says that if person A and person B are distinguishable in one CC, X, and then there is another CC, X’, in which they are indistinguishable, then there is some person C in X’-X that is distinguishable from person A. This axiom is similar in spirit to van Benthem’s *upward difference* in that it ensures that if there is a contrast/distinction in one comparison class, the existence of a contrast is maintained in all the larger CCs. *Minimal difference* says that if, at the finest level of granularity, you would make a distinction between two individuals with respect to the semantic denotation of a predicate, then they are not indistinguishable at that level of granularity. MD is similar in spirit to van Benthem’s *downward difference* because it allows us to preserve contrasts down to the smallest comparison classes.

(48) For all predicates P_1, all interpretations $[\![\cdot]\!]_{M,g}$, all $X \subseteq D$,

(1) **Contrast preservation (CP)**: For all $X' \subseteq D$, and $a_1, a_2 \in X$, if $X \subset X'$ and $a_1 \nsim^{X}_{P_1} a_2$ and $a_1 \sim^{X'}_{P_1} a_2$, then $\exists a_3 \in X' - X : a_1 \nsim^{X'}_{P_1} a_3$.

(2) **Minimal difference (MD)**: For all $a_1, a_2 \in D$, if $[\![a_1 >_{P_1} a_2]\!]_{M,g,X} = 1$, then $a_1 \nsim^{\{x,y\}}_{P_1} a_2$.

In summary, I propose that the definitions of the $\sim_P$s, $\sim_Q$s and $\sim_R$s are constrained by the axioms in Table 2.

3.3. Consequences

The analysis laid out in Table 2 has certain immediate consequences for the relationship between tolerant, classical, and strict denotations of absolute predicates. In particular, the total axiom prohibits members of the predicate’s classical extension from being indifferent from members of the predicate’s classical anti-extension. A consequence of this result is that the strict interpretation of a total predicate *always* coincides with its classical interpretation, as shown by Theorem 11.

Theorem 11. *For all total predicates Q_1 (i.e., predicates that satisfy the AAA and the relevant axioms in Table 2), all models M, all $X \subseteq D$ and all $a_1 \in X$, $[\![Q_1(a_1)]\!]^s_X = [\![Q_1(a_1)]\!]_X$.*

Proof. Let $X \subseteq D$ and $a_1 \in X$. Clearly, by the AAA and the fact that the strict denotation is always a subset of the classical denotation (cf. Cobreros et al., 2012, Lemma 1), $[\![Q_1(a_1)]\!]_X = 1$ if $[\![Q_1(a_2)]\!]^s_X = 1$. Now, suppose $[\![Q_1(a_1)]\!]_X = 1$ and suppose for a contradiction that $[\![Q_1(a_1)]\!]^s_X = 0$. So, by Definition 10, there is some $a_2 \in X$ such that $a_2 \sim^X_{Q_1} a_1$ and $[\![Q_1(a_2)]\!]_X = 0$. However, by the total axiom (TA), $a_2 \not\sim^X_{Q_1} a_1$. $\perp$ Therefore $[\![Q_1(a_1)]\!]^s_X = 1$. ■

Furthermore, since the classical and strict denotations of total predicates are identical across all contexts, their associated scales (i.e., the denotations of their comparative predicates) are identical, and since the classical scales associated with total predicates are trivial, so are their strict scales.

Corollary 12. *For all total predicates Q_1, $>^s_{Q_1} = >_{Q_1}$.*

Although a total predicate's strict and classical interpretations coincide, this is not necessarily the case for its tolerant interpretation. In fact, as proved by Burnett (2012b), taken as a whole, the constraints on the $\sim_Q$ relations in Table 2 allow the tolerant interpretation of the total comparative predicate to be a possibly non-trivial strict weak order. This result is stated as Theorem 13.

Theorem 13. *(Burnett, 2012b). If Q_1 is a total predicate (i.e., its interpretation is subject to the AAA and the constraints proposed in Table 2), $>^t_{Q_1}$ is a (possibly non-trivial) strict weak order.*

The partial axiom (PA), on the other hand, prohibits elements of a partial predicate's classical anti-extension from being indifferent from elements of its classical extension. Thus, with the partial class of predicates, it is the strict interpretation of the comparative that is a possibly non-trivial strict weak order (cf. Theorem 14; Burnett, 2012b), and it is the tolerant interpretation of the predicate that coincides with its classical interpretation (Theorem 15).

Theorem 14. *(Burnett, 2012b). If R_1 is a partial predicate (i.e., its interpretation is subject to the AAA and the constraints proposed in Table 2), $>^s_{R_1}$ is a (possibly non-trivial) strict weak order.*

Theorem 15. *For all partial predicates R_1 (i.e., predicates that satisfy the AAA and the relevant axioms in Table 2), all models M, all $X \subseteq D$ and all $a_1 \in X$, $[\![R_1(a_1)]\!]^t_X = [\![R_1(a_1)]\!]_X$.*

Proof. Let $X \subseteq D$ such that $a_1 \in X$. Suppose that $[\![R_1(a_1)]\!]_X = 1$. Then by the reflexivity of $\sim^X_{R_1}$ and Definition 9, $[\![R_1(a_1)]\!]^t_X$. Now, suppose $[\![R_1(a_1)]\!]^t_X = 1$ and, for a contradiction, suppose $[\![R_1(a_1)]\!]_X = 0$. So, by Definition 9, there is some $a_2 \in X$ such that $\mathbf{a_2} \sim^{\mathbf{X}}_{\mathbf{R_1}} \mathbf{a_1}$ and $[\![R_1(a_2)]\!]_X = 1$. But, by the partial axiom (PA), $\mathbf{a_2} \not\sim^{\mathbf{X}}_{\mathbf{R_1}} \mathbf{a_1}$. $\perp$ So $[\![R_1(a_1)]\!]_X = 1$. ■

And we find a parallel corollary with tolerant and classical scales being identical for partial predicates.

Corollary 16. *For all partial predicates R_1, $>^t_{R_1} = >_{R_1}$.*

Finally, the fact that total and partial predicates are both predicted to be associated with a single non-trivial scale has further consequences for the distribution of certain modifiers that were introduced in section 2, such as *loosely speaking* and *strictly speaking*. Recall that *loosely speaking* appears only with total adjectives like *dry* and seems to pick out the individuals in a comparison class that are approximately Q without actually being Q. This modifier is very bizarre with partial adjectives, as shown in (49-b).

(49) a. This towel is loosely speaking dry (but it is not completely dry).
b. ?This towel is loosely speaking wet.

On the other hand, *strictly speaking* appears only with partial predicates and seems to pick out those individuals in the comparison class who are classically R but not strictly R.

(50) a. ?This towel is strictly speaking dry.
b. This towel is strictly speaking wet (but it is not really wet).

An intuitive analysis of these modifiers within DelTCS (shown in (51)) would be one in which *loosely speaking* picks out the complement of a predicate's classical denotation in its tolerant denotation and *strictly speaking* picks out the complement of a predicate's strict denotation in its classical denotation.

(51) For all total and partial predicates Q, all $X \subseteq D$ and all $n \in \{t, s, c\}$,

a. $[\![\text{loosely speaking Q}]\!]^n_X = [\![Q]\!]^t_X - [\![Q]\!]_X$.
b. $[\![\text{strictly speaking Q}]\!]^n_X = [\![Q]\!]_X - [\![Q]\!]^s_X$.

Since the strict interpretation of a total predicate is exactly its classical interpretation (Theorem 11), the denotation of *strictly speaking dry* will always be empty and therefore we correctly predict that total adjectives should be impossible with this modifier. Furthermore, since, by Theorem 15, the tolerant interpretations of partial adjectives are exactly their classical denotations, the denotation of an expression like *loosely speaking wet* will always be the null set. We therefore correctly predict that use of *loosely speaking* with partial predicates should be bizarre. I therefore conclude that the general DelTCS framework and the proposed analysis of absolute predicates within it has great potential to account for the distribution and interpretation of hedges and expressions of precision in the adjectival domain.[13]

In the next section, I show that the logical analysis within DelTCS that was presented in this section predicts the reasoning patterns described in section 2.

4. Scalarity, maximality and evaluativity results

We saw in section 2 that all classes of gradable predicates satisfy the scalarity argument: if A is taller/emptier/wetter than B and B is tall/empty/wet, then A is tall/empty/wet. That relative predicates satisfy scalarity is already given in the basic delineation analysis of Klein (1980) and van Benthem (1982). This fact is proved as the (meta-)Theorem 17.

Theorem 17. *For all models M, comparison classes $X \subseteq D$, relative predicates P_1, $a_1, a_2 \in X$,*

- *If $[\![a_1 >_{P_1} a_2]\!]_X = 1$ and $[\![P_1(a_2)]\!]_X = 1$, then $[\![P(a_1)]\!]_X = 1$.*

Proof. Suppose that $[\![a_1 >_{P_1} a_2]\!]_X = 1$ and $[\![P_1(a_2)]\!]_X = 1$ to show that $[\![P_1(a_1)]\!]_X = 1$. Suppose for a contradiction that $[\![P_1(a_1)]\!]_X = 0$. So $[\![\neg P_1(a_1)]\!]_X = 1$ and since $[\![P_1(a_2)]\!]_X = 1$, by Definition 5, $[\![a_2 >_{P_1} a_1]\!]_X = 1$. However, since by Theorem 7

$>_{P_1}$ is a strict weak order, it is asymmetric. Therefore $[\![a_2 >_{P_1} a_1]\!]_X = 0$. $\perp$ As there are bots at the end of a number of proofs now, I am starting to think that they are deliberate ? but I still wish to check. Therefore $[\![P_1(a_1)]\!]_X = 1$. ■

As shown in the proof of Theorem 17, the crucial property of the denotation of the comparative predicates that allows scalarity to go through is the asymmetry of $>_{P_1}$, which comes from it being a strict weak order. We saw in the previous section that the tolerant interpretation of the total comparative predicate is a possibly non-trivial strict weak order and the strict interpretation of the partial comparative predicate also has this property. Thus, the scalarity argument will go through on $[\![\cdot]\!]^t$ and $[\![\cdot]\!]^s$ for total and partial predicates respectively. Recall furthermore that the classical and tolerant (for partial) and strict (for total) interpretations of absolute predicates were also strict weak orders, albeit trivial ones. Therefore, we predict that absolute predicates should satisfy scalarity under all interpretations.[14] This being said, I suggest that the triviality of the strict interpretation of the total comparative (i.e., *straighter than*) can explain the uneasy feeling that authors such as Unger, Lewis, Kennedy and McNally have about comparatives like (52).

(52) If stick A is straighter than stick B and stick B is straight, then stick A is straight.

a. If $[\![a_1 >_{Q_1} a_2]\!]^s_X = 1$ and $[\![Q_1(a_2)]\!]^s_X = 1$, then $[\![Q_1(a_1)]\!]^s_X = 1$.

The statement is true because the antecedent is false: it is impossible in this framework for a stick to be strictly (or classically) straight and yet be less straight than some other stick. In other words, we can show that total predicates satisfy the maximality argument on the strict interpretation of the positive form of the predicate (and on any interpretation of the comparative, where ϵ indicates the empty string).

Theorem 18. *For all models M, comparison classes $X \subseteq D$ and total predicates Q_1, $a_1, a_2 \in D$, and $m \in \{t, s, \epsilon\}$,*

- *If $[\![a_1 >_{Q_1} a_2]\!]^m_X = 1$ then $[\![Q_1(a_2)]\!]^s_X = 0$.*

Proof. Case 1: $(m = \epsilon)$. Suppose that $[\![a_1 >_{Q_1} a_2]\!]_X = 1$ to show that $[\![Q_1(a_2)]\!]^s_X = 0$. Since $a_1 >_{Q_1} a_2$, by Definition 5, there is some $X' \subseteq D$ such that $[\![Q_1(a_2)]\!]_{X'} = 0$. By the AAA, $[\![Q_1(a_2)]\!]_X = 0$, so by Definition 10, $[\![Q_1(a_2)]\!]^s_X = 0$. ✓
Case 2: $(m = s)$. Suppose that $[\![a_1 >_{Q_1} a_2]\!]^s_X = 1$ to show that $[\![Q_1(a_2)]\!]^s_X = 0$. Since $>^s_{Q_1} = >_{Q_1}$ (cf. Corollary 12), by parity of reasoning with case 1, $[\![Q_1(a_2)]\!]^s_X = 0$. ✓
Case 3: $(m = t)$. Suppose $[\![a_1 >_{Q_1} a_2]\!]^t_X$ to show $[\![Q_1(a_2)]\!]^s_X = 0$. Since $a_1 >^t_{Q_1} a_2$, by Definition 9, there is some $X' \subseteq D$ such that $[\![Q_1(a_2)]\!]^t_X = 0$. Also by Definition 9, since $[\![Q_1(a_2)]\!]^t_X = 0$, $[\![Q_1(a_2)]\!]_X = 0$. Finally, by Theorem 11, $[\![Q_1(a_2)]\!]^s_X = 0$. ✓ ■

Note however that maximality does not go through on the tolerant interpretation of the positive form of a total predicate, even when we consider the non-trivial tolerant interpretation of the comparative ($>^t_{Q_1}$). This fact is shown through the countermodel in (53). In section 2, I argued that this was indeed the pattern that we see in the data.

(53) Let M be a t-model such that $D = \{a_1, a_2, a_3\}$. Let Q_1 be a total predicate such that $m(Q_1, \{a_1, a_2, a_3\}) = \{a_1\}$. So $[\![Q_1(a_1)]\!]_{\{a_1,a_2\}} = 1$ and $[\![Q_1(a_2)]\!]_{\{a_1,a_2\}} = 0$. By MD, $a_1 \not\sim^X_{Q_1} a_2$, so $\mathbf{a_1 >^t_{Q_1} a_2}$. Now suppose $a_1 \sim^{\{a_1,a_2,a_3\}}_{Q_1} a_2$. So $\mathbf{[\![Q_1(a_2)]\!]^t_{\{a_1,a_2,a_3\}} = 1}$, even though $a_1 >_{Q_1} a_2$.

Similar countermodels can be constructed for all interpretations of relative and partial adjectives, and I leave this as an exercise for the reader.

Finally, I turn to evaluativity. In section 2 I argued that partial predicates were evaluative only on their tolerant interpretation. Indeed, as shown in Theorem 19, this is a (meta)theorem of DelTCS.

Theorem 19. *For all models M, comparison classes $X \subseteq D$ and partial predicates R_1, $a_1, a_2 \in D$ and $m \in \{t, s, \epsilon\}$,*

- *If* $[\![a_1 >_{R_1} a_2]\!]^m_X = 1$ *then* $[\![R_1(a_1)]\!]^t_X = 1$.

Proof. Case 1: $(m = \epsilon)$. Suppose $[\![a_1 >_{R_1} a_2]\!]_X = 1$ to show that $[\![R_1(a_1)]\!]^t_X = 1$. Since $a_1 >_{R_1} a_2$, there is some $X' \subseteq D$ such that $[\![R_1(a_1)]\!]_{X'} = 1$. By the AAA, $[\![R_1(a_1)]\!]_X = 1$. So, by Definition 9, $[\![R_1(a_1)]\!]^t_X = 1$. ✓
Case 2: $(m = s)$. Suppose $[\![a_1 >_{R_1} a_2]\!]^s_X = 1$ to show that $[\![R_1(a_1)]\!]^t_X = 1$. Suppose for a contradiction that $[\![R_1(a_1)]\!]^t_X = 0$. Then, by Definition 9, $[\![R_1(a_1)]\!]_X = 0$. By the AAA, $[\![\mathbf{R_1(a_1)}]\!]_{\mathbf{D}} = \mathbf{0}$. Since $a_1 >^s_{R_1} a_2$, by Definition 10, there is some $X' \subseteq D$ such that $[\![R_1(a_1)]\!]^s_{X'} = 1$. Also by Definition 10, $[\![R_1(a_1)]\!]_{X'} = 1$. So, by the AAA, $[\![R_1(a_1)]\!]_D = 1$. $\perp$ So $[\![R_1(a_1)]\!]^t_X = 1$. ✓
Case 3: $(m = t)$. Suppose $[\![a_1 >_{R_1} a_2]\!]^t_X = 1$ to show that $[\![R_1(a_1)]\!]^t_X = 1$. By Corollary 16, $>^t_{R_1} = >_{R_1}$. Therefore since $a_1 >^t_{R_1} a_2$, $a_1 >_{R_1} a_2$. Thus, by Definition 5, $[\![R_1(a_1)]\!]_X = 1$ and by Definition 9 $[\![R_1(a_1)]\!]^t_X = 1$. ✓ ■

On the other hand, as shown in (54), the strict interpretation of a partial adjective is not evaluative: A can be wetter than B without A being really wet. Similar countermodels can be constructed to show that relative adjectives are not evaluative on any interpretation.

(54) Let M be a t-model such that $D = \{a_1, a_2, a_3\}$. Let R_1 be a partial predicate such that $m(R_1, \{a_1, a_2, a_3\}) = \{a_1, a_2\}$. So $[\![R_1(a_2)]\!]_{\{a_2,a_3\}} = 1$ and $[\![R_1(a_3)]\!]_{\{a_2,a_3\}} = 0$. By MD, $a_3 \not\succ^X_{R_1} a_2$, so $\mathbf{a_2} >^{\mathbf{s}}_{\mathbf{R_1}} \mathbf{a_3}$. Now suppose $a_3 \sim^{\{a_1,a_2,a_3\}}_{R_1} a_2$. So $[\![\mathbf{R_1(a_2)}]\!]^{\mathbf{s}}_{\{\mathbf{a_1},\mathbf{a_2},\mathbf{a_3}\}} = \mathbf{0}$, even though $a_2 >_{Q_1} a_3$.

In parallel with this, total predicates are not evaluative on their tolerant interpretation: a_1 can be higher on the 'approximately empty' scale than a_2 without counting as tolerantly empty in the context. I leave the appropriate countermodel to the reader. Interestingly, the strict interpretations of total predicates satisfy the *evaluativity* proposition in some restricted cases, namely when the comparative is also interpreted strictly (or classically). This is shown in Theorem 20.

Theorem 20. *For all models M, all total predicates Q_1, all $X \subseteq D$ and $a_1, a_2 \in X$,*

- *If* $[\![a_1 >_{Q_1} a_2]\!]^s_X = 1$, *then* $[\![Q_1(a_1)]\!]^s_X = 1$.

Proof. Suppose $[\![a_1 >_{Q_1} a_2]\!]^s_X = 1$. Since, by Corollary 12, $>^s_{Q_1} = >_{Q_1}$, $a_1 >_{Q_1} a_2$. By the AAA, $[\![Q_1(a_1)]\!]_X = 1$ and by Theorem 11, $[\![Q_1(a_1)]\!]^s_X = 1$. ■

However, if we are interested in the non-trivial interpretation of the comparative, its tolerant interpretation, then the strict interpretation of the positive stops being evaluative. This is to say that, unlike partial predicates whose tolerant interpretations under every way of interpreting the comparative (cf. Theorem 19), arguments like (55) are not valid for total predicates, since a_1 could be only tolerantly, not strictly, Q_1. Thus, total predicates are not predicted to be evaluative in the way that partial predicates are.

(55) *False:* For all models M and all $X \subseteq D$, if $[\![a_1 >_{Q_1} a_2]\!]^t_X = 1$ then $[\![Q_1(a_1)]\!]^s_X = 1$.

Countermodel: Let M be a t-model such that $D = \{a_1, a_2, a_3\}$ and let Q_1 be a total predicate such that $m(Q_1, \{a_1, a_2, a_3\}) = \{a_3\}$. Now suppose that $a_3 \sim^{\{a_1,a_2,a_3\}}_{Q_1}$

Table 3. Reasoning patterns associated with relative, partial and total adjectives.

ARGUMENT	RELATIVE		TOTAL		PARTIAL	
	$[\![\cdot]\!]^t$	$[\![\cdot]\!]^s$	$[\![\cdot]\!]^t$	$[\![\cdot]\!]^s$	$[\![\cdot]\!]^t$	$[\![\cdot]\!]^s$
Scalarity	✓	✓	✓	✓	✓	✓
Maximality	×	×	×	✓	×	×
Evaluativity	×	×	×	×	✓	×

a_1, so $[\![Q_1(a_1)]\!]^t_{\{a_1,a_2,a_3\}} = 1$. Suppose furthermore that $a_3 \not\sim^{\{a_1,a_2,a_3\}}_{Q_1} a_2$, so $[\![Q_1(a_2)]\!]^t_{\{a_1,a_2,a_3\}} = 0$. Therefore, by Definition 9, $[\![\mathbf{a_1} >_{\mathbf{Q_1}} \mathbf{a_2}]\!]^{\mathbf{t}}_{\{\mathbf{a_1},\mathbf{a_2},\mathbf{a_3}\}} = \mathbf{1}$. However, since $[\![Q_1(a_2)]\!]_{\{a_1,a_2,a_3\}} = 0$, $[\![\mathbf{Q_1}(\mathbf{a_2})]\!]^{\mathbf{s}}_{\{\mathbf{a_1},\mathbf{a_2},\mathbf{a_3}\}} = \mathbf{0}$.

Finally, I mentioned in section 2 that total predicates appear to be ‘pseudo-evaluative’ in the sense that their subjects have to be at least somewhat close to the strict/classical denotation of the positive form of the adjective. I suggest that a possible source of this ‘pseudo-evaluativity’ comes from the definition of the tolerant comparative in Definition 9. In particular, in order for an individual to truthfully appear in the subject position of a tolerantly interpreted comparative construction, they do not have to tolerantly satisfy the positive form of the predicate in the context; however, there must be *some* (perhaps very large) comparison class in which they do. Thus, I hypothesise that the existence of such a context is what creates the feeling of evaluativity that we observe with total comparative predicates.

5. Conclusion

In conclusion, in this paper I showed how the *Delineation TCS* non-classical logical framework, which had previously been proposed for other purposes, can model the scalarity, maximality and evaluativity of three classes of gradable adjectives. The empirical properties that were argued for in this paper are summarised in Table 3.

I showed that the analysis presented within *Delineation TCS* predicts exactly the observed reasoning patterns from axioms dealing with the nature of contextual variation and judgements of indifference. I therefore conclude that this framework can shed new light on the logical analysis of gradable adjectives and the interaction between tolerance relations, context-sensitivity, and scalar reasoning in natural language.

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Notes

1. It is common in the linguistics literature to make a distinction between gradable adjectives which have commonly used comparative forms (e.g., *taller*, *more intelligent*, etc.) and non-scalar

adjectives which are not very natural in the comparative construction (e.g., *more atomic*, *more geographical*, *primer*, etc.).

2. This distinction also has consequences for the properties of the scales associated with the respective predicates. These 'scale structure' consequences are examined within the DelTCS framework in Burnett (2012b).
3. The judgements of which arguments can be formed with which predicates that are presented in this paper are those of the author, which have been checked with a handful of other native speakers. Ideally it would be desirable to investigate these patterns from a large-scale experimental perspective; however, since I have yet to find speakers who are in disagreement with the inferences presented here, I take the data presented in this section to be representative of the reasoning patterns of at least a sizeable group of English speakers.
4. See also Sassoon and Toledo (2011) for a discussion of these conflicting data that comes to a similar conclusion to the one presented in this paper. Given these examples, one might wonder whether the different word orders in (17) create the different felicity judgements. Since it is possible for me (and other speakers that I have consulted) to interpret both sentences as contradictions or not, depending on the level of precision, I suggest that, while word order may make one interpretation more salient over another for some speakers, English syntax is not solely responsible for the interpretative variation in (17).
5. In the discussion of the interpretation of total predicates above, I likened the strict interpretation of a total adjective to a 'precise' use of this adjective and its tolerant interpretation to its 'imprecise' use. This is consistent with current terminology in the literature (c.f., for example, Lewis, 1979; Lasersohn, 1999; Récanati, 2010), but when we consider the strict and tolerant interpretations of relative and partial adjectives like *tall* and *wet*, perhaps a better way of describing the difference between $[\![P]\!]^s$ and $[\![P]\!]^t$ is as a difference between the set of individuals that are clearly or really P (the strict interpretation) and the set that includes those individuals that are borderline P (the tolerant interpretation).
6. The system in Cobreros et al. (2012) was proposed to model the puzzling properties of vague language with relative predicates like *tall*; however, I suggest that the results in this paper and in Burnett (2013) show that it has a natural application to modelling similar effects with absolute adjectives.
7. Throughout this paper, I will use $\sim_P$ and $>_P$ to refer to both the indifference and comparative relations in the model and the indifference and comparative predicates in the language. I do not believe that this will cause confusion. Note furthermore that I will use $\sim_P^X$ to notate an indifference relation in a model at a comparison class X, whereas the comparison classes are not syntactically represented in the language.
8. For example, suppose that in the CC $\{a_1, a_2\}$, $[\![P(a_1)]\!]_{\{a_1,a_2\}} = 1$ and $[\![\neg P(a_2)]\!]_{\{a_1,a_2\}} = 1$. So $a_1 >_P a_2$. And suppose moreover that, in the larger CC $\{a_1, a_2, a_3\}$, $[\![P(a_2)]\!]_{\{a_1,a_2,a_3\}} = 1$ and $[\![\neg P(a_1)]\!]_{\{a_1,a_2,a_3\}} = 1$. So $a_2 >_P a_1$. But clearly, natural language comparatives do not work like this: if John is {taller, fatter, wider, etc.} than Mary, Mary cannot also be {taller, fatter, wider, etc.} than John. In other words, $>_P$ must be asymmetric.
9. The definitions of *irreflexivity, transitivity* and *almost connectedness* are as follows:

Definition 21. ***Irreflexivity***. *A relation $>$ is irreflexive iff there is no $x \in D$ such that $x > x$.*

Definition 22. ***Transitivity.*** *A relation $>$ is transitive iff for all $x, y, z \in D$, if $x > y$ and $y > z$, then $x > z$.*

Definition 23. ***Almost connectedness***. *A relation $>$ is almost connected iff for all $x, y \in D$, if $x > y$, then for all $z \in D$, either $x > z$ or $z > y$.*

10. For example, one cannot be taller than oneself; therefore $>_{tall}$ should be irreflexive. Also, if John is taller than Mary, and Mary is taller than Peter, then we know that John is also taller than Peter. So $>_{tall}$ should be transitive. Finally, suppose John is taller than Mary. Now consider Peter. Either Peter is taller than Mary (same height as John or taller) or he is shorter than John (same height as Mary or shorter). Therefore, $>_{tall}$ should be almost connected.
11. This approach has recently been incarnated in, for example, Kennedy and McNally (2005), Récanati (2010), van Rooij (2011b), and Burnett (2012b).

12. The definition of $\geq_P$ that is featured in the next two constraints is as follows. We first define an equivalence relation $\approx_P$:

Definition 24. ***Equivalent***. *($\approx$) For an interpretation $[\![\cdot]\!]_{M,g,X}$, a predicate P, $a_1, a_2 \in D$:*

(1) $a_1 \approx_P a_2$ *iff* $[\![a_1 >_P a_2]\!]_{M,g,X} = 0$ *and* $[\![a_2 >_P a_1]\!]_{M,g,X} = 0$.
(2) $a_1 \approx^t_P a_2$ *iff* $[\![a_1 >_P a_2]\!]^t_{M,g,X} = 0$ *and* $[\![a_2 >_P a_1]\!]^t_{M,g,X} = 0$.
(3) $a_1 \approx^s_P a_2$ *iff* $[\![a_1 >_P a_2]\!]^s_{M,g,X} = 0$ *and* $[\![a_2 >_P a_1]\!]^s_{M,g,X} = 0$.

Now we define $\geq_P$:

Definition 25. ***Greater than or equal***. *($\geq$) For an interpretation $[\![\cdot]\!]_{M,g,X}$, a predicate P, $a_1, a_2 \in D$:*

(1) $a_1 \geq_P a_2$ *iff* $[\![a_1 >_P a_2]\!]_{M,g,X} = 1$ *or* $a_1 \approx_P a_2$.
(2) $a_1 \geq^t_P a_2$ *iff* $[\![a_1 >_P a_2]\!]^t_{M,g,X} = 1$ *or* $a_1 \approx^t_P a_2$.
(3) $a_1 \geq^s_P a_2$ *iff* $[\![a_1 >_P a_2]\!]^s_{M,g,X} = 1$ *or* $a_1 \approx^s_P a_2$.

13. Note, however, that the application of adverbs *strictly speaking* and *loosely speaking* still needs to be restricted to the class of absolute adjectives, i.e., those adjectives without context-sensitive classical denotations, since *loosely/strictly speaking* is impossible with relative adjectives (i.e., *John is loosely/strictly speaking tall*).

14. Interestingly, as discussed in Burnett (2012b), with the constraints proposed for indifference relations with relative adjectives in Table 2, the tolerant and strict interpretations of relative comparative predicates are not quite strict weak orders because transitivity fails for some degenerate cases. However, Burnett (2012b) shows that these relations still satisfy a version of van Benthem's *no reversal* and relative predicates satisfy scalarity even on their tolerant and strict interpretations.

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