Formal Language Theory 2

1 Closure properties of Reg

Is Reg closed under $\cap, \cup, -$?

(1) For all $L_1, L_2 \in Reg$,

a. $L_1 \cap L_2 \in Reg$
b. $L_1 \cup L_2 \in Reg$
c. $L_1 - L_2 \in Reg$

Consider intersection. Suppose $L_1, L_2 \in Reg$ to show $L_1 \cap L_2 \in Reg$.

Proof idea. Since $L_1, L_2 \in Reg$, there are autonoma A_1, A_2 such that $L(A_1) = L_1$ and $L(A_2) = L2$.

- Now we find an algorithm for creating a new FSA $A_1 \cap A_2$: $L(A_1 \cap A_2) = L_1 \cap L_2$.

Here is the algorithm:

Let $L(A_1) = L_1$ and let $L(A_2) = L_2$.

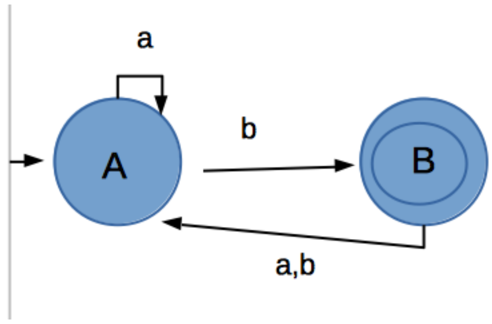
So $A_1 : \langle Q_1, \Sigma, \delta_1, q_1, F_1\rangle$ and $A_2 : \langle Q_2, \Sigma, \delta_2, q_2, F_2\rangle$.

Let $A_{1\cap 2} : \langle Q, \Sigma, \delta, q_0, F\rangle$, such that

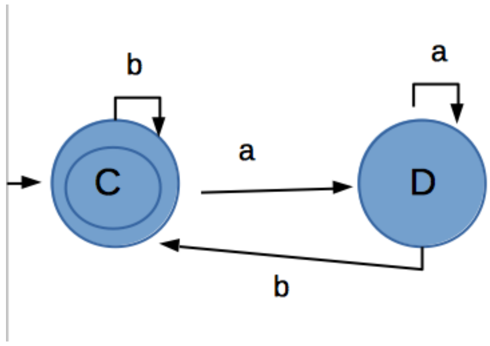
1. $Q = \{\langle r_1, r_2\rangle : r \in Q_1 \text{ and } r_2 \in Q_2\rangle$
2. $q_0 = \langle q_1, q_2\rangle$
3. $F_1 \times F_2$
4. δ : for all $\langle r_1, r_2\rangle \in Q$ and $a \in \Sigma$, $\delta(\langle\langle r_1, r_2\rangle, a\rangle) = \langle \delta_1(r_1, a), \delta_2(r_2, a)\rangle$.

Example. Show that the intersection of the languages generated by FSAs A_1 and A_2 is also regular.

(2) A_1



(3) A_2



What are closure properties of regular languages good for?

- (Among other things) helping us establish which languages are regular.
- If $L_1, L_2 \in Reg$, then $L_1 \cap L_2 \in Reg$.

Suppose $L_1 \in Reg$ and $L_1 \cap L_2 \notin Reg$. Then $L_2 \notin Reg$.

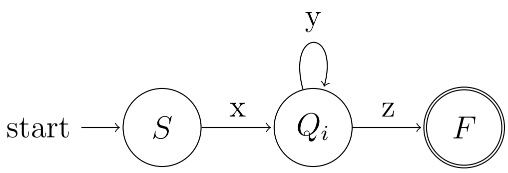
- We will use this reasoning to show that ENG $\notin Reg$.
- We pick a language that we know is regular, intersect it with English to produce another language, which we then show to $\notin Reg$.

1.1 How to show $L \notin Reg$?

We use a **Pumping Lemma**.

Consider an infinite $L \in Reg$. So L is accepted by an FSA.

- Since L is infinite, the FSA has a loop.



A accepts xyz, $xyyz, xyyyz$, etc. In other words, for all $n \geq 0$, A accepts xy^nz.

- There is a particular **structure** in L(A).

Définition 1.1 ***Pumping Lemma.*** *If $L \in Reg$, then there are strings $x, y, z \in \Sigma^*$ such that $y \neq \epsilon$ and $xy^nz \in L$, for all $n \geq 0$.*

Consider a^nb^n. Is $a^nb^n \in Reg$?

- If $a^nb^n \in Reg$, it has the structure xy^nz.
- So where is the y^n part?

(4) Cases

a. Some number of *a*s followed by some number of *b*s. $x(ab)^nz$
b. Some number of *a*s. xa^nz
c. Some number of *b*s. xb^nz

Pump these patterns:

(5) $x(ab)^n z$

a. ab n = 0
b. aabb n = 1
c. aababb $\notin a^n b^n$ n = 2

(6) $xa^n z$

a. ab n = 0
b. aab $\notin a^n b^n$ n = 1

(7) $xb^n z$

a. ab n = 0
b. abb $\notin a^n b^n$ n = 1

Therefore $a^n b^n \notin Reg$.

1.2 What about Eng(lish)?

Consider the subset of Eng:

(8) L_1

a. A famous prof hired a famous prof.
b. A famous prof a famous prof hired, hired a famous prof.
c. A famous prof a famous prof a famous prof hired hired hired a famous prof.

L_1 has the structure (a famous prof)^n(hired)^n a famous prof, so $L_1 \notin Reg$.

- $L_1 \subset Eng$.
- $L_1 \subset$ (a famous prof)*(hired)* a famous prof (L_2).
- $L_2 \cap Eng = L_1$.

We know that Reg is closed under intersection, $L_2 \in Reg$ and $L_1 \notin Reg$. Therefore, $Eng \notin Reg$ □

More generally: languages which have unbounded structures requiring matching are not regular.