Formal Learning Theory 2

1 Some positive results

1.1 $\mathcal{L}_{co-1}$ is identifiable

(1) a. Let $\mathcal{L}_{co-1}$ be the class of languages $\{\Sigma^* - \{x\} | x \in \Sigma^*\}$
b. For any number $k \geq 0$, let $\mathcal{L}_{co-k}$ be the class of languages $\{\Sigma^* - L | |L| = k\}$

A grammar for a language in $\mathcal{L}_{co-1}$ could be regarded as a "filter" represented by a rule of the form (where x is any string):

(2) $^*x$

A grammar for any language in $\mathcal{L}_{co-k}$ in one of the classes of $\mathcal{L}_{co-k}$ could be given by exactly k filters of this kind.

Théorème 1.1 *$\mathcal{L}_{co-1}$ is identifiable.*

Proof: Consider any enumeration of Σ^* and let ϕ be defined as follows:

- For any sequence of strings $T[i]$,
$$\phi(T[i]) = x*$$
, where $*x$ is the first string in the enumeration that does not occur in $T[i]$.

Now we show that, given any text T for any language $L \in \mathcal{L}_{co-1}$, ϕ identifies T.

- Since $L \in \mathcal{L}_{co-1}$, L is defined by some filter $*x$. Since there are just finitely many elements that occur before x, there will be some finite point i such that T[i] contains all those elements.
- At that point, by the definition of ϕ, $\phi[T(i)] = *x$.
- Furthermore, it is easy to see that at all later points $j > i$, $\phi(T[j]) = *x$ because x will never occur in the text.

Generalization:

Théorème 1.2 *For any k, $\mathcal{L}_{co-k}$ is identifiable.*

Proof: One learner for $\mathcal{L}_{co-k}$ is the function that, at each point, conjectures that the first k elements of Σ^* not seen so far are the ones excluded.

2 Locking sequences

(3) **Locking sequence for a learner ϕ on a language L:** A finite sequence such that no extension of that sequence with elements of L can make ϕ change its mind.

Définition 2.1 *Given a learner ϕ that identifies a language L, a sequence t is said to be a* ***locking sequence*** *for ϕ and L iff*

1. *t is a finite sequence of expressions and #.*
2. *$content(t) \subseteq L$*
3. *$L(\phi(t)) = L$*
4. *For every finite sequence s such that $content(ts) \subseteq L$, $\phi(ts) = \phi(t)$.*

Théorème 2.1 *If L is any (recursively enumerable) language and ϕ identifies L, then there is a locking sequence t for ϕ and L. (Blum and Blum 1975)*

3 Negative results

Théorème 3.1 *The class $\mathcal{L}_{co-1} \cup \mathcal{L}_{co-2}$ is not identifiable.*

Proof: Suppose for contradiction that this class is identifiable. Then there is some learner ϕ that identifies the class.

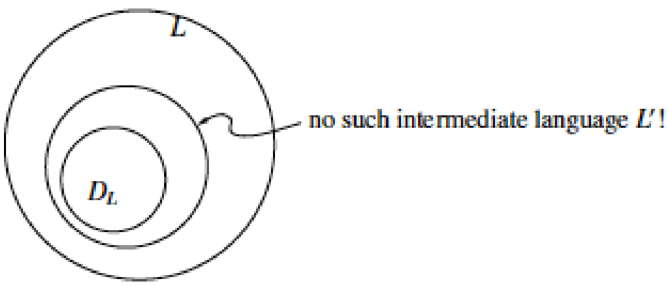
- Consider $L \in \mathcal{L}_{co-1}$. By Theorem 2.1, there is a locking sequence t for L and ϕ.
- Since L is infinite and content(t) is finite, there is an $x \in L - content(t)$.
- Consider any text T' for the language for the language $L' = (L - \{x\}) \in \mathcal{L}_{co-2}$. Since $content(T') \subset L$, and since t is a locking sequence for ϕ and L, $\phi(tT')$ converges on a grammar for L.
- But this is incorrect since $L \neq content(tT')$. So ϕ does not identify this text, so it does not identify $(L - \{x\})$. $\perp$ □

(4) **Fact:** If $\mathcal{L}$ is not identifiable, then no superset of $\mathcal{L}$ is identifiable.

- If the superset of $\mathcal{L}$ were identifiable, some learner would identify all of its languages. . . including all those in $\mathcal{L}$.

4 Angluin's subset theorem

Théorème 4.1 *Let $\mathcal{L}$ be a set of (recursively enumerable) languages. Then $\mathcal{L}$ is identifiable iff for all $L \in \mathcal{L}$ there is a finite distinguished subset D_L such that no language L' that includes D_L is properly included in L.*



Proof: ⇒ Suppose ϕ identifies $\mathcal{L}$. Then for any $L \in \mathcal{L}$ there is a locking sequence t for ϕ and L. Since locking sequences are finite, so is $content(t)$. This finite subset of L will distinguish it: that is, (we can show) that for all $L' \in L$, if $content(t) \subseteq L'$, then $L' \not\subset L$.

⇐ Suppose that for every $L \in \mathcal{L}$ there is a finite distinguished subset D_L such that no language L' that includes D_L is properly included in L.

- Suppose that we have an enumeration of all possible grammars. For each $L \in L$, we fix a particular finite $\mathrm{D}_L \subseteq L$ such that for all $L' \in L$ if $D_L \subseteq L'$ then $L' \not\subset L$.
- Then for every text T, define $\phi(T[i])$ to be the first grammar G in the enumeration such that $D_{L(G)} \subseteq content(T[i]) \subseteq L(G)$, if there is one. Otherwise, let $\phi(T[i])$ be the first grammar in the enumeration.
- (We can show) that ϕ identifies $\mathcal{L}$. □

(5) **Corollary of Angluin's theorem:** Let $\mathcal{L}$ be a collection of (recursively enumerable) languages such that:

1.$\mathcal{L}$ contains an infinite language L_∞, and

2.For every $L_0 \mathcal{L}_\infty$, there is some $L \in \mathcal{L}$ such that $L_0 \subseteq L \subset L_\infty$.

Then, $\mathcal{L}$ is not identifiable.

Proof: Assume $\mathcal{L}$ satisfies (i) and (ii). By (ii), every finite subset L_0 of L_∞ is such that for some $L \in \mathcal{L}$, $L_0 \subseteq L \subset L_\infty$, and so Angluin's theorem applies immediately. □

5 Gold's theorem

(6) **Main result:** No strict superset of $\mathcal{L}_{fin}$ (the set of finite languages) is identifiable. (Gold 1967)
Proof: Immediate from (5). □

This theorem has been hugely influential in cognitive science and linguistics.

- There are many misunderstandings about what exactly it says.
- Argument against empiricism.

5.1 Misunderstandings

(Deacon 1997, 127-128), cited in Johnson (2004).

> [Gold] provided a logical proof which concluded that, without explicit error correction, the rules of a logical system with the structural complexity of a natural language grammar could not be inductively discovered, even in theory. What makes them unlearnable, according to this argument, is not just their complexity but the fact that the rules are not directly mapped to the surface forms of sentences. The result is that sentences exhibit hierarchic syntactic structures, in which layers of transformations become buried and implicit in the final product, and in which structural relationships between different levels can often produce word-sequence relationships that violate relationships that are appropriate within levels. From the point of view of someone trying to analyze sentence structure (such as a linguist or a young language learner), this has the effect of geometrically multiplying the possible hypothetical rules that must be tested before discovering the 'correct' ones for the language

- Structural complexity has nothing to do with Gold learning...

Elman et al. 1996 (p.343).

> Interestingly, sentences with relative clauses possess exactly the sort of structural features which may make (according to Gold 1967) a language unlearnable?

- Learning classes of Ls is what is relevant (a single language is trivially learnable, in the Gold sense).

5.2 Argument against empiricism

Johnson's summing up of the arguments in favour of rationalism:

(7)
a. If there are no constraints on language acquisition, then either children have access to negative data or natural languages are unlearnable.
b. If they exist, the constraints in question must be innate.
c. Children don?t have access to negative data.
d. Natural languages are learnable.
e. ∴ There are innate constraints on language acquisition.