Assignment 3: Propositional Logic

Due May 15th, 2019

Question 1

Compute stepwise each of the following:

(1)
a. $AF(P_5 \vee \neg P_5)$
b. $AF(\neg(P_5 \vee P_6))$
c. $AF(P_6 \leftrightarrow P_6)$
d. $AF((P_1 \rightarrow P_9) \vee (P_1 \rightarrow \neg P_9))$

(Hint: Replace the formulas containing $\rightarrow$ and $\leftrightarrow$ with the ones that define them.)

Question 2

Establish the claims below by exhibiting an assignment of truth values to the atomic formulas in the left hand formula which make it true and the right hand formula false. We write $\not\models$ for does not entail.

(2)
a. $(\neg P_4 \vee P_7) \not\models (\neg P_7 \vee P_4)$
b. $((P_4 \vee P_7)\ \&\ (P_8 \vee P_7)) \not\models (P_4 \vee P_8)$
c. $((P_1 \& P_2) \vee P_3) \not\models (P_1\ \&\ (P_2 \vee P_3))$

Question 3

For each pair of formulas below show that they are logically equivalent if they are and exhibit a line of their truth table at which they differ if they are not.

(3)
a. i. $((P\ \&\ Q) \vee (\neg P\ \&\ \neg Q))$ ii. $(P \leftrightarrow Q)$
b. i. $(P \rightarrow Q)$ ii. $(\neg Q \rightarrow \neg P)$
c. i. $(P \rightarrow Q)$ ii. $((P\ \&\ Q) \leftrightarrow P)$
d. i. $((P\ \&\ Q) \vee P)$ ii. P
e. i. $(P\ \&\ (Q \vee R))$ ii. $((P\ \&\ Q) \vee (P\ \&\ R))$

Question 4

Prove the coincidence lemma.

(4) **The coincidence lemma.** For all $\phi \in PROP$ and all models v and u, if $v(P_n) = u(P_n)$ for all atomic formulas occurring in ϕ, then $v^*(\phi) = u^*(\phi)$.

(Hint: look at the structure of the proof that all formulas only have a finite number of AFs that we did in class.)