Class 7: Game Theory 1

1 Introduction

What is game theory?

- A tool to study economic, political and biological phenomena.
- The study of models that depict aspects of the interaction of agents/decision makers.

A **model** is an abstraction we use to understand our observations and experiences.

- Understanding: perceiving relationships between situations, isolating principles that apply to a range of problems sot hat we can incorporate new situations into our way of thinking.
- The basic assumptions that underlie the theory are that agents pursue well-defined exogenous objectives (they are rational) and take into account their knowledge or expectations of other agents' behavior (they reason strategically).

1.1 Theory of rational choice

(1) **Theory of Rational Choice (general):** An agent chooses the best action according to her preferences, among all the actions available to her.

- No qualitative restriction is placed on the decision-maker's preferences.

A basic model with two components:

1. A set A consisting of all the actions that, under some circumstances, are available to the decision-maker.
2. A specification of the agent's preferences.

Preferences are represented by a (ordinal) *payoff function* u from actions to $\mathbb{N}$ such that:

(2) $u(a) > u(b)$ iff the decision-maker prefers a to b.

A person is faced with the choice of three vacation packages, to Montreal, Rome, and Venice. She prefers the package to Montreal to the other two, which she regards as equivalent.

- Her preferences between the three packages are represented by any payoff function that assigns the same number to both Rome and Venice and a higher number to Montreal.
  1. We can set $u(\text{Montreal}) = 1$ and $u(\text{Rome}) = u(\text{Venice}) = 0$,
  2. $u(\text{Montreal}) = 10$ and $u(\text{Rome}) = u(\text{Venice}) = 1$,
  3. $u(\text{Montreal}) = 0$ and $u(\text{Rome}) = u(\text{Venice}) = \text{-}2$.

(3) **Theory of Rational Choice (specific):** The action chosen by an agent is at least as good, according to her preferences, as every other available action.

2 Strategic Games, Complete Information

(4) **Strategic game (with ordinal preferences)**
A strategic game (with ordinal preferences) consists of:

- a set of players (N = $\{p_1, \ldots p_n\}$).
- for each player, a set of actions ($\langle A_1, \ldots A_n \rangle$).
- for each player, preferences over the set of action profiles ($A_1 \times \ldots \times A_n$).

- Action profiles are tuples of all possible combinations of individual choices.
- For two players i, j, with two sets of actions A_i, A_j, this games action profiles are $A_i \times A_j$. So $u(\langle a_i, a_j \rangle) = n$ for some $n \in \mathbb{N}$.

Game theory distinguishes different kinds of games, traditionally classified along two dimension:

1. whether the agents' choices are simultaneous or in sequence,
2. whether all agents have complete or incomplete information.

Static vs dynamic

- Games where players move simultaneously are called static games (alt.: strategic games);
- games where players move in sequence are called dynamic games (alt.: sequential games).

Complete vs incomplete information

- We say that a player has complete information in a game if she knows all the decision relevant details except for the play of other players.
- A player who knows the action choices of her opponents has perfect information.

Standardly, game theory assumes players to be imperfectly informed, so that individual decision making crucially depends on conjectures about other players' behavior. It is in this sense that games model decisions in interactive situations.

(5) **Prisoner's dilemma**
Two suspects in a major crime are held in separate cells. There is enough evidence to convict each of them of a minor offense, but not enough evidence to convict either of them of the major crime unless one of them acts as an informer against the other (confess). If they both stay quiet (deny), each will be convicted of the minor offense and spend one year in prison. If one and only one of them confesses, she will be freed and used as a witness against the other, who will spend four years in prison. If they both confess, each will spend three years in prison.

Strategic game:

- Players: N = {player 1, player 2}
- A = {confess, deny}
- Preferences over action profiles: $\{u_1, u_2\}$

(6) Constraints on us
a. u_1(confess, deny) $>$ u_1(deny, deny) $>$ u_1(confess, confess) $>$ u_1(deny, confess)

b. u_2(deny, confess) > u_2(deny, deny) > u_2(confess, confess) > u_2(confess, deny)

	c	*d*
c	2,2	0,3
d	3,0	1,1

Figure 1.1: The prisoner's dilemma

- Strategic game with complete information. (everyone moves at the same time; both players know the payoffs of the others).

(7) **Bach or Stravinsky** Two people wish to go out together. Two concerts are available: one of music by Bach, and one of music by Stravinsky. One person prefers Bach and the other prefers Stravinsky. If they go to different concerts, each of them is equally unhappy listening to the music of either composer

example set up: (complete information; static game).

	Bach	*Stravinsky*
Bach	2, 1	0, 0
Stravinsky	0, 0	1, 2

Figure 16.1 *Bach or Stravinsky?* (*BoS*) (Example 16.2).

3 Solution Concepts

This game is really just a model of the situation and does not specify what the agents in fact do, (or should do, if we are being prescriptive).

(8) **Solution concept**
An algorithm that specifies how the game is/should be played.

- Specifies the idealized behavior of agents in the situation that is modelled by the game.

(9) **Nash equilibrium (informal)**
An action profile a* with the property that no player i can do better by choosing an action different from a_i^* , given that every other player j adheres to a_j^*.

(10) **Nash equilibrium (formal)**
an action profile a^* such that for all $i \in N$ there is no $a_i \in A_i$ for which

$$(a^*_{-i}, a_i) \prec_i a^*)$$

(a^*_{-i}, a_i) is the tuple that results from replacing the i-th component of a^* with a_i.

3.1 Prisoner's dilemma

The action pair (confess, confess) is a Nash equilibrium because

- given that player 2 chooses confess, player 1 is better off choosing confess than deny.

- given that player 1 chooses confess, player 2 is better off choosing confess than deny .

No other action profile is a Nash equilibrium:

- (deny, deny) does not satisfy NE because when player 2 chooses deny, player 1's payoff to confess exceeds her payoff to deny.
- (confess, deny) does not satisfy NE because when player 1 chooses confess, player 2's payoff to confess exceeds her payoff to deny.
- (deny, confess) does not satisfy NE because when player 2 chooses confess, player 1's payoff to confess exceeds her payoff to deny.

Unintuitive result!
How to interpret nash equilibria?

- Prescriptive (how to get the best outcome).
- Descriptive: this is how people make decisions.
  - Experimental results. Mixed (refs if you want).
  - Size of payoffs, setting (face to face).

3.2 For future work. . .

- How to choose between multiple equilibria. (refining)
- Rationality and links with reasoning:
  - Our algorithm for describing how the games are played involves: find Nash Equilibria. But we haven't said anything about how they are found. Simple comparison.
  - Here, we are modelling human behaviour, but the solution concept makes no reference to reasoning.
  - Especially pertinent for games of partial information (signalling games), since we think that pragmatic interpretation is driven by reasoning.